Seismic Slope Safety Assessment

Analytical Approach

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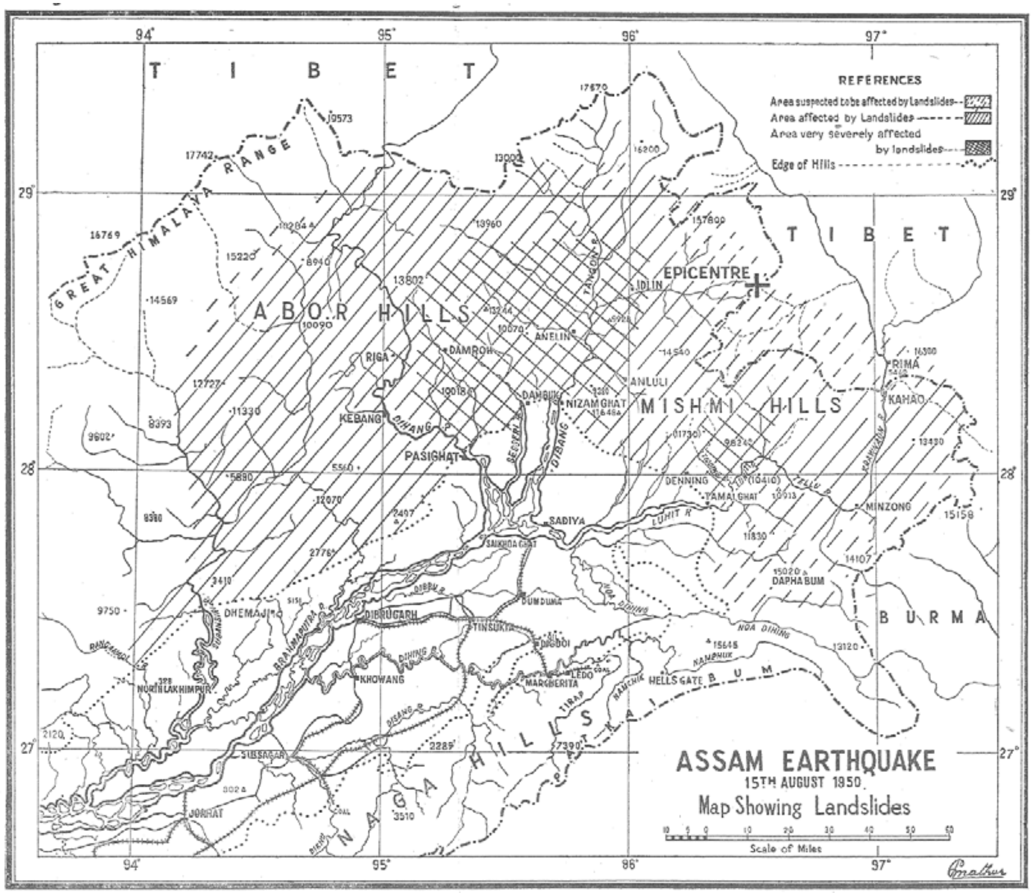


Example of slope failure in Assam earthquake of 15 August 1950 in India. [After Mathur (1953)]. About 30000 km^2 area was affected.

Example of slope failure in Assam earthquake of 15 August 1950 in India. The figure shows the damaged valley of the river Simen north of Dibrugarh [After Gee (1953)]

Seismic Slope Safety Assessment
Analytical Approach

- Triggering factors
  - Natural
    - Earthquakes
    - Rainfall
    - Toe-cutting by river erosion
  - Man-Made
    - Toe cutting for Road Building, Houses
    - Loading on the Ridge
    - Reservoir Building

Seismic Slope Safety Assessment
Simplified Approach

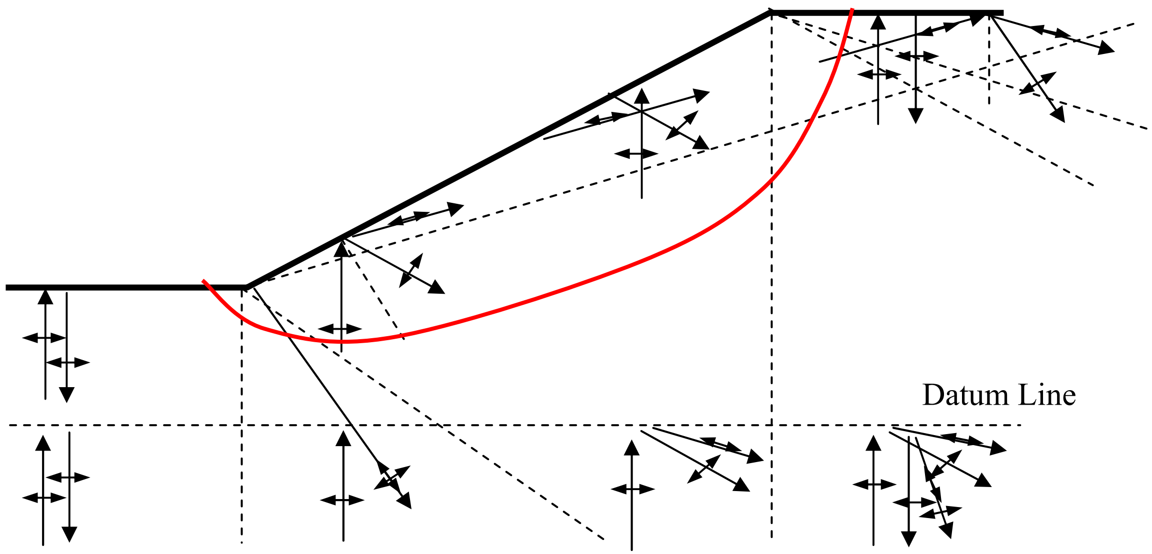
- Inertia forces generated by the earthquakes
- Resistance of the slope to the static forces plus the inertia forces
- Consequences if the resistance is less than the applied forces

Seismic Slope Safety Assessment

Inertia forces

- Understand the wave mechanics
- Boundary conditions

Seismic Slope Safety Assessment



Wave characteristics in slopes

SV wave traveling from below. The long arrows show the wave direction, the short arrows show the particle motion. The particle motion at any point will be the combined effect of all waves. (Sarma&Irakleidis,2008)

Seismic Slope Safety Assessment

Inertia Forces

- It is understood that the inertia forces represent an average value over a limited mass.
- Therefore, the acceleration (Force/Mass) is an average acceleration assumed constant over the mass.
- The acceleration is a time-history.

Seismic Slope Safety Assessment
Ground Motion

- The ground motion is usually represented by a “design” peak ground acceleration in the design codes.
- $k_m g$ = Design peak ground acceleration
- Transient Phenomenon
- Lasts for a very short duration
- Only one or two Pulses containing the peak motion which lasts for a fraction of a second

Seismic Slope Safety Assessment

Resistance to the forces

- Soil has a Limit Strength and therefore, resistance is limited.
- If the resistance is not sufficient, then slip surface forms in the slope.
- Measure of safety of the slope
  - Factor of safety
  - Critical acceleration

Seismic Slope Safety Assessment

Limit Strength

Undrained conditions. Excess Pore water pressure generated by the earthquake has no time to dissipate during the earthquake.

– Undrained Strength, c_u
– Effective strength with pore water pressures

Seismic Slope Safety Assessment

Soil Properties

- Undrained Shear Strength- C_u
- Depends on many factors – most important are Normal stress on the failure plane and the pore water pressure conditions

Seismic Slope Safety Assessment
Soil Properties

- Soil Strength – Mohr- Coulomb Failure Criterion
- Effective strength parameters
  - Friction angle
  - Cohesion

  $\tau = c' + \sigma' \tan\phi'$

  τ = Shear Strength

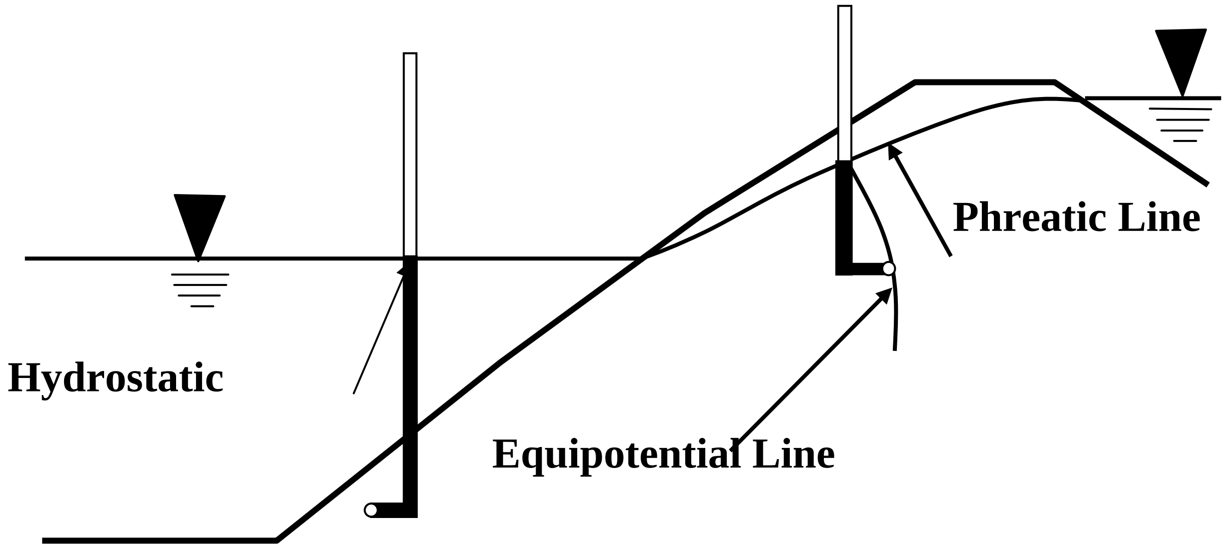
  c' = Cohesion

  σ' = Effective normal stress = Total normal stress-pore water pressure

  ϕ' = Friction angle

Seismic Slope Safety Assessment
Pore water Pressure

Hydrostatic and Pressure due to flow



Pore water pressures under static (non-seismic) conditions.

Seismic Slope safety Assessment
Pore water Pressure

- Excess Pore water pressure due to seismic loading
  - Depends on many factors
  - Increases with cycles of loading
  - Dissipates slowly after the earthquake
  - Dissipation depends on the permeability of the soil

Seismic Slope Safety Assessment

Analysis Technique

- Determine
  - Geometry of Slope
  - Soil Properties
  - Pore Water Pressure (Static +Earthquake)
- Apply a method of analysis

Seismic Slope Safety Assessment

Analysis Technique

- Measure of Safety
  - Factor of Safety
    - F = Available average Shear Strength/Mobilised Shear Stress Measured along a critical slip surface
  - Critical Acceleration
    - $K_c g$= Acceleration required to make factor of safety equal to one measured along a critical slip surface

Seismic Slope Safety Assessment
Analysis Technique

- Factor of Safety Or Critical Acceleration
  - $F > 1$ or $k_c > k_m$ ------- Safe
  - $F = 1$ or $k_c = k_m$ ------- Imminent failure. A slip surface is formed
  - $F < 1$ or $k_c < k_m$ -------The mass above the slip surface slides down hill. The motion can be stopped by
    - Change of loading condition
    - Increase of Strength along the slip surface
    - Rearrangement of the Geometry of the slide

Seismic Slope Safety Assessment

Simplified Analysis Technique

- Limit Equilibrium Analysis
  - Simple
  - Easy to understand
  - Looks at failure situation only

Seismic Slope Safety Assessment

Analysis Technique

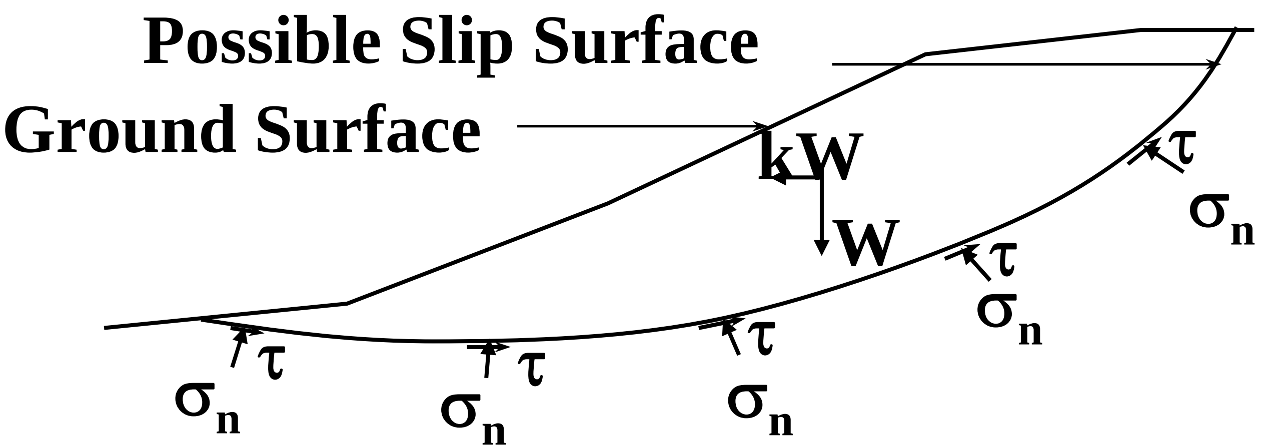
- Limit Equilibrium Technique
  - Define a possible slip surface
  - Define the loads (self weight, Inertia Load, pore water pressure in the slope etc)
  - Determine Either the Factor of Safety
  - Or, the Critical Acceleration.
  - Find the critical surface with minimum F or minimum k_c

Seismic Slope Safety Assessment
Limit Equilibrium Technique

- The aim: For a given possible slip surface and for a given loading condition, find a state of stress (normal and shear components) on the slip surface so that the free body contained within the slip surface and the free surface is in equilibrium.

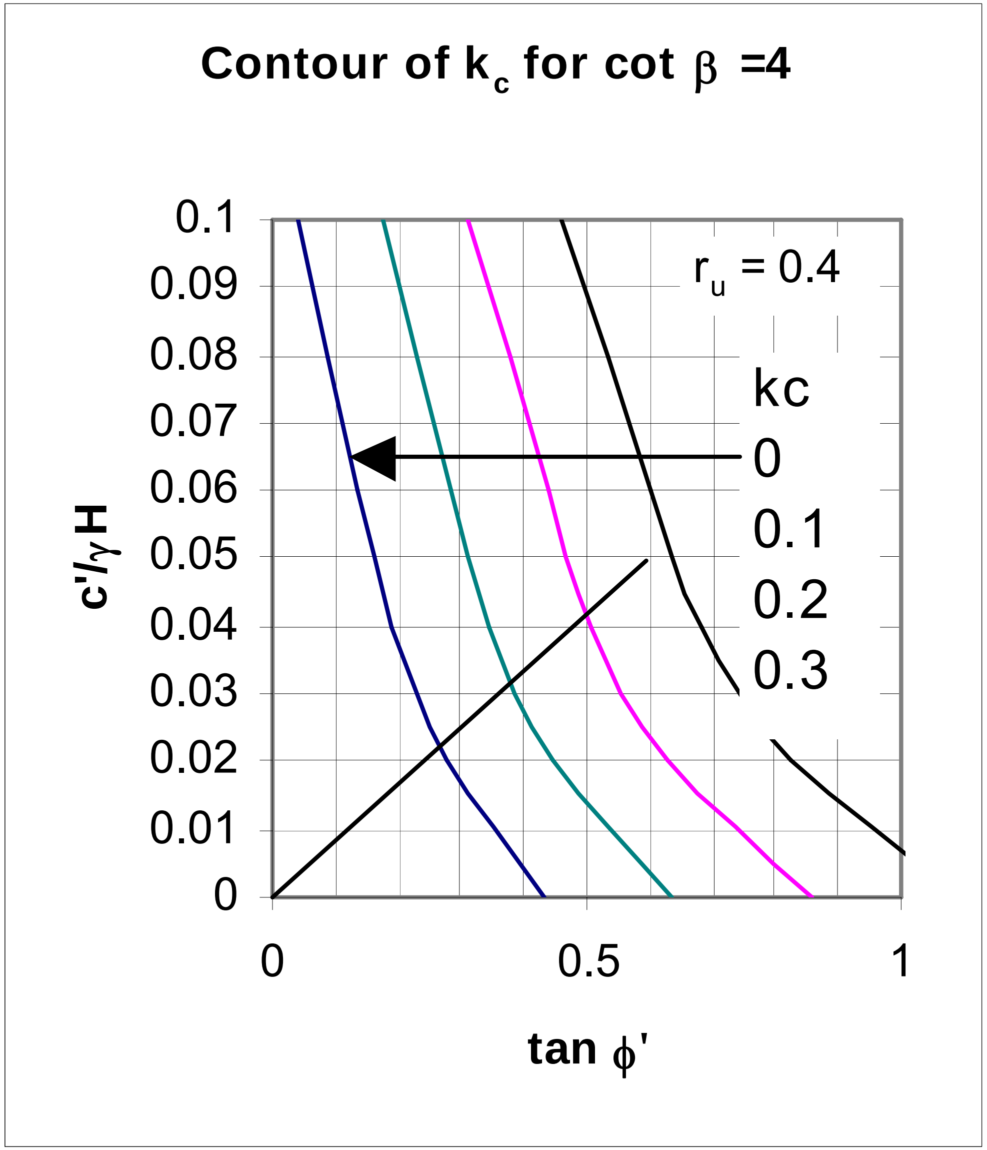
Seismic Slope Safety Assessment

LIMIT EQUILIBRIUM TECHNIQUE



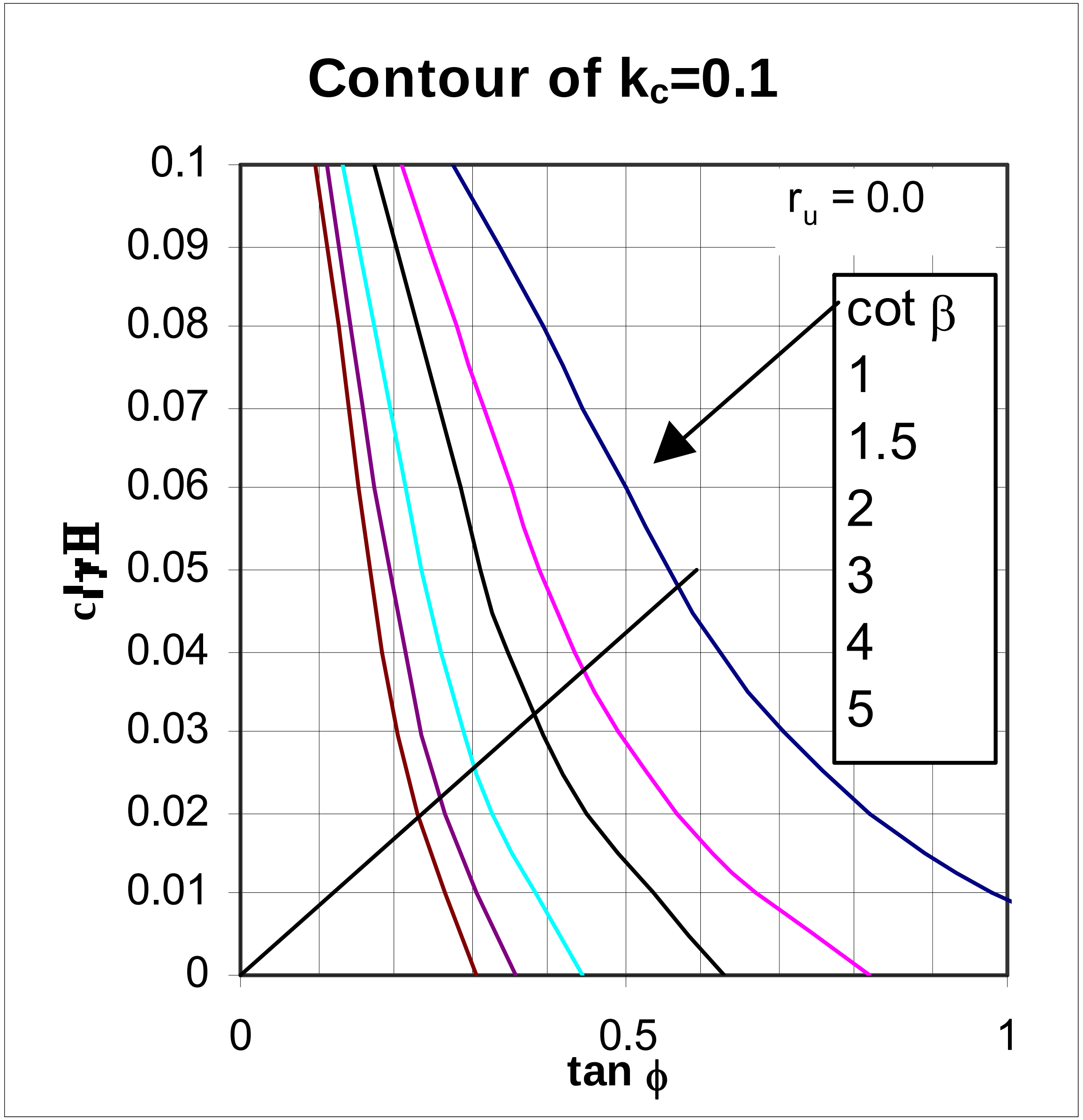
<u>Schematic Slope and a possible slip surface</u>

(W is the weight of the soil above the slip surface, kW is a horizontal load, σ_n is the normal stress and τ is the shear stress on the slip surface. The horizontal load may represent the inertia of the soil mass due to an earthquake acceleration kg. Water in the slope, if any, will produce pore water pressure in the soil, which will effect the strength but not the equilibrium stresses.)



Homogeneous slope (1/4 Gradient)

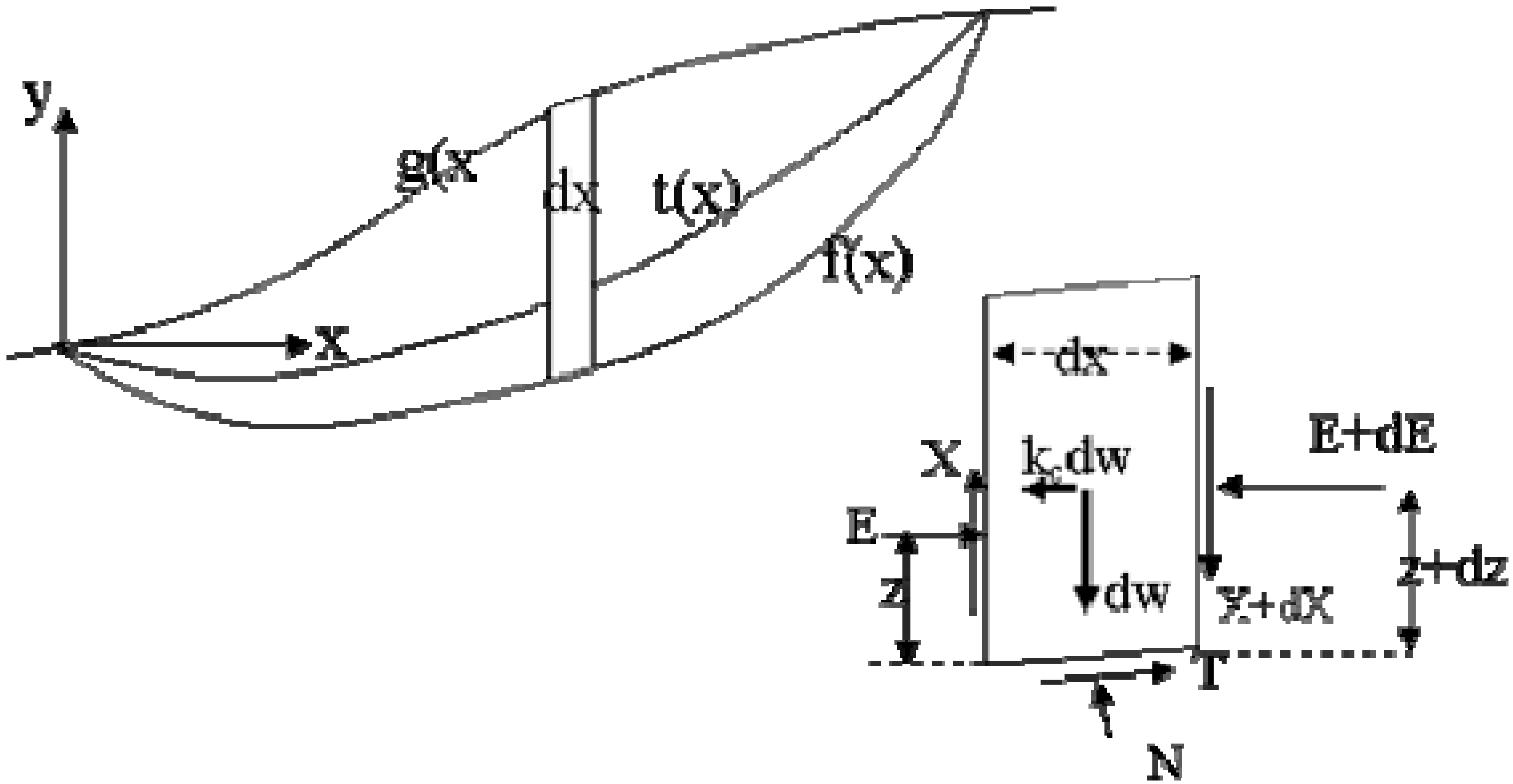
PWP parameter $r_u = u/\gamma H$



Contour of k_c=0.1
r_u = 0.0
cot β
1
1.5
2
3
4
5
c/γH
tan φ
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.5
1

Seismic Slope Safety Assessment

Limit Equilibrium Technique



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Limit Equilibrium Technique

$$E''z + E'[2z'+f'-(1+af')/(a-f')] + E[z''+f'']$$

$$= 1/(a-f')[(1+af')k_c w'-(a-f')w'-c'(1+f'2)]- k_c w'' (g-f)/2 - k_c w' (g'-f')/2$$

$$= gk_c[(g-f)(1+af')/(a-f')-(g'-f')(g-f)]-(g-f)g-(1+f'2)/(a-f')c'$$

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Limit Equilibrium Technique

- For a given slip surface f(x), and an assumed thrust line t(x), a function E(x) exist for any solution of k_c. Or, for an assumed E(x), a solution k_c can be found.
- Within this technique, infinite solutions are possible.
- Therefore, we need controls to restrict the number of solutions.
- Various available methods are different ways of applying controls.
- The solutions must be acceptable. Implied stresses from the solution must not violate soil strength or kinematics.

Seismic Slope Safety Assessment
Limit Equilibrium Technique

- Acceptability Criterion:
  - Normal stresses on the slip surface and the interslice thrusts are compressive. (Soil cannot take tensions)
  - Directions of shear on the slip surface and on the interslice boundaries must be kinematically acceptable so that a downhill sliding can take place given the chance.
  - The factor of safety on any plane inside the slide mass must be greater than or equal to one (Soil has limit strength)

Seismic Slope Safety Assessment

Limit Equilibrium Technique

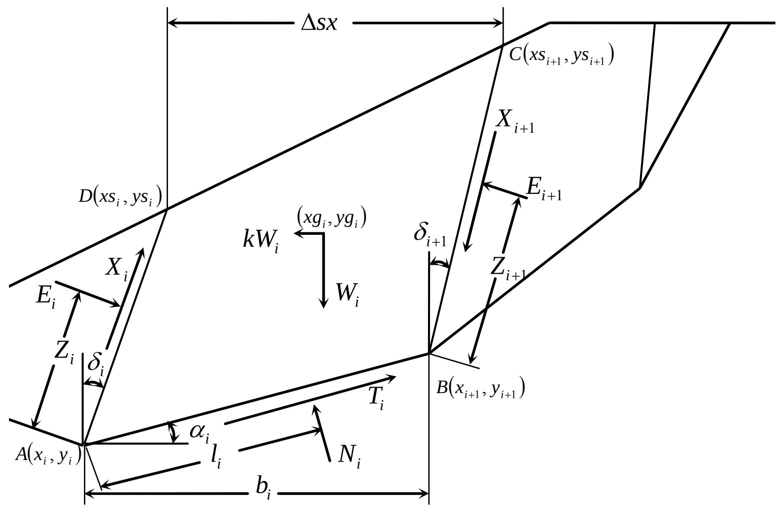
- **Question**: Is there an acceptable solution k_c for an assumed slip surface?

- **Alternative Question**: Is there an acceptable slip surface for any assumed k_c?

Seismic Slope Safety Assessment

Limit Equilibrium Technique

- Method of Inclined Slices, Sarma(1979)- Solves for k_c for a given slip surface. Then, starting from the first slice
- Using Equations:
  - Vertical equilibrium
  - Horizontal equilibrium
  - Limit strength on slip surface
  - Limit strength on shear surface
- Determines Unkonwns:
  - N_i, T_i, E_{i+1}, X_{i+1}

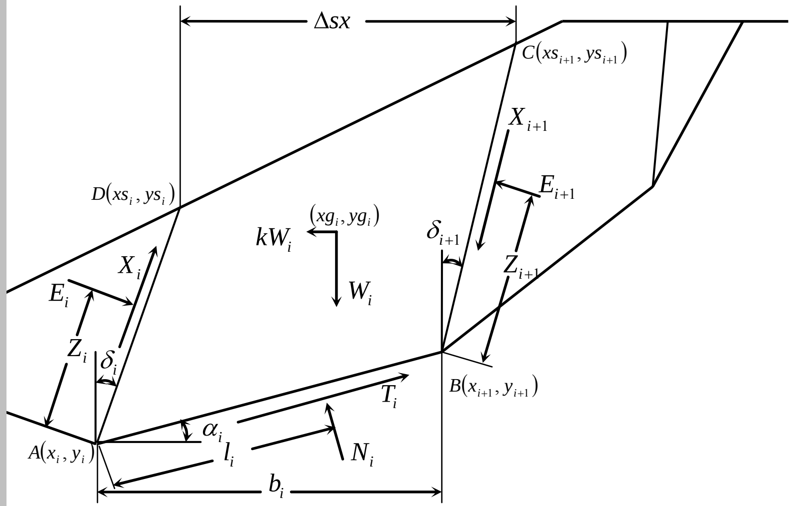


Seismic Slope Safety Assessment

Enhanced Limit Equilibrium Technique

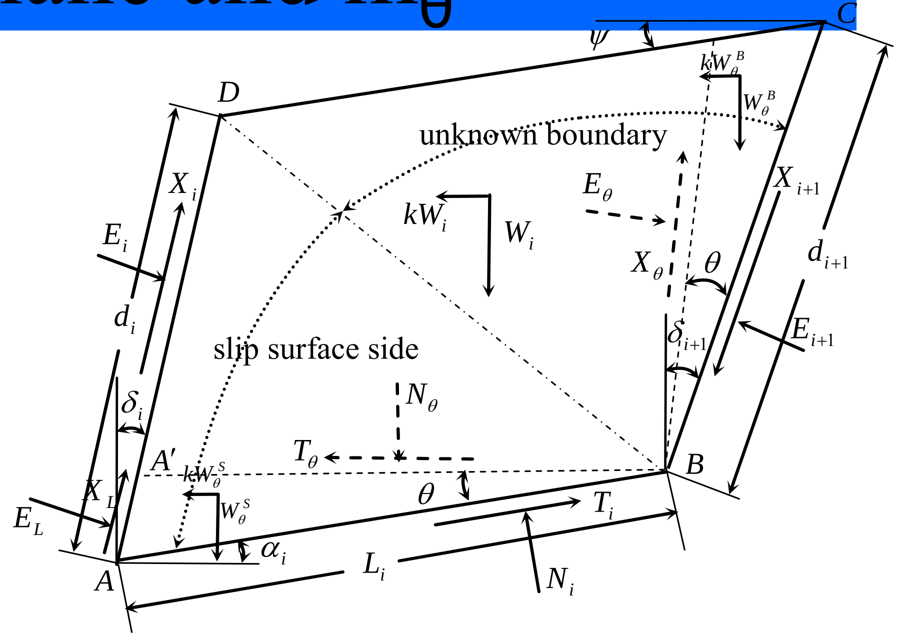
Sarma & Tan (2006)

- Assume k_c is known but the geometry of the ith slice is unknown
- Two Extra Unkonwns:
  - α_i, δ_{i+1}
- Two Extra Equations needed:



Enhanced Limit Equilibrium Technique Internal plane and m_θ

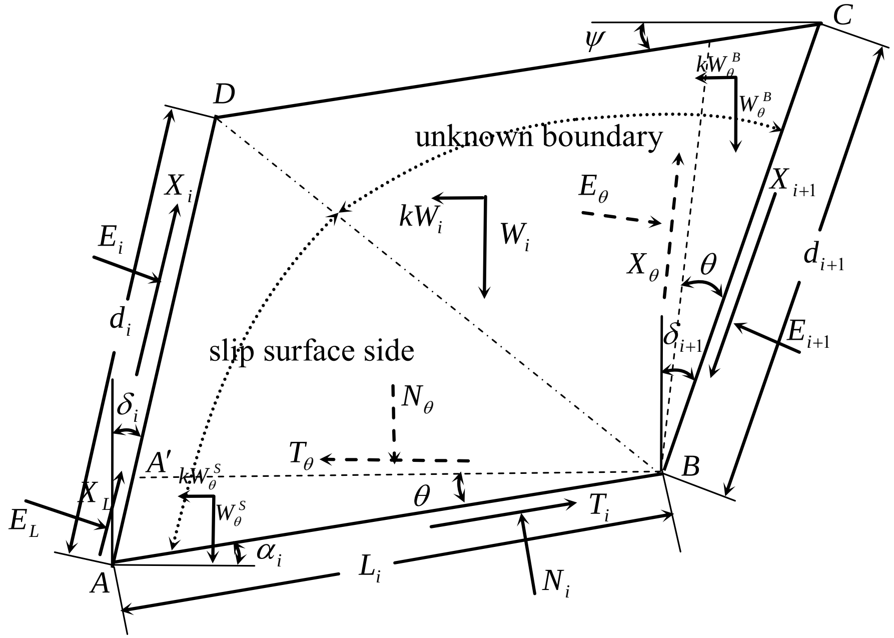
- the slice is divided into two segments
- The forces on an internal plane can be obtained from force equilibrium
- m_θ, the reciprocal of the factor of safety on an internal plane, can also be found:



Enhanced Limit Equilibrium Technique
Internal plane and m_θ

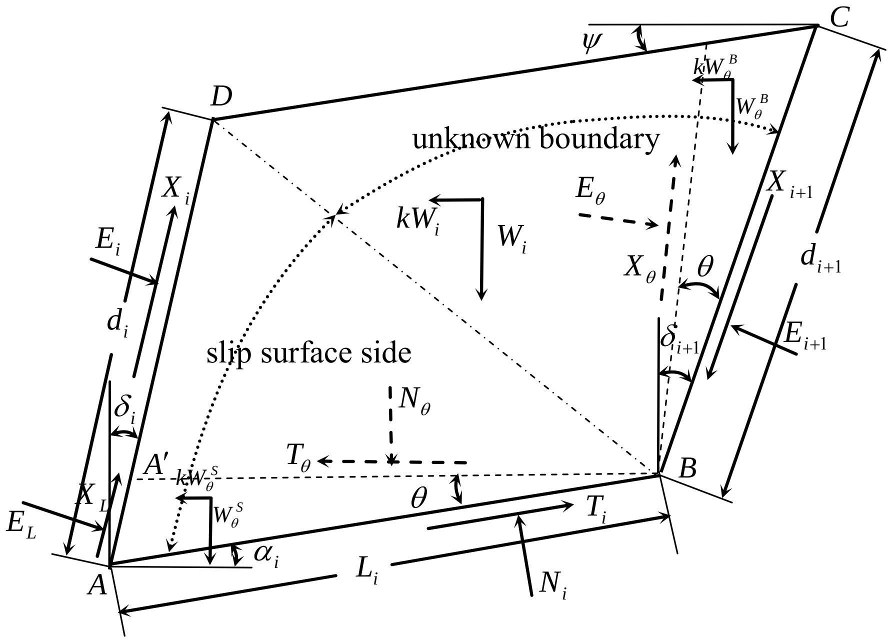
- Consider two planes close to the slip surface, one inside the slide mass and the other outside.

The factor of safety on these planes must be greater than or equal to one.



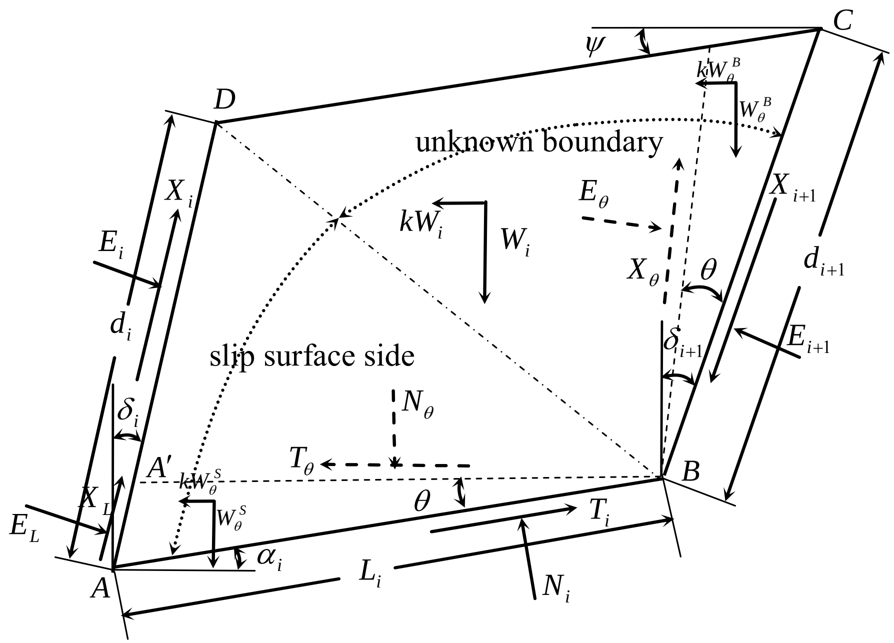
Enhanced Limit Equilibrium Technique Internal plane and m_θ

- Therefore, on the slip surface, the reciprocal of the factor of safety, m_θ, must be a <u>maximum</u> value equal to one.
- This gives one equation.



Enhanced Limit Equilibrium Technique Internal plane and m_θ

- Similar condition also applies to the Interslice Boundary end
- This condition gives a second equation.
- Now, we can solve for the geometry unknowns and the force unknowns for the slice for a given k_c



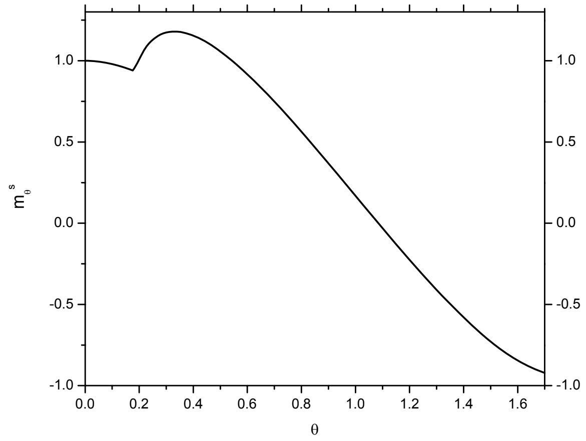
Enhanced Limit Equilibrium Technique
Internal plane and m_θ

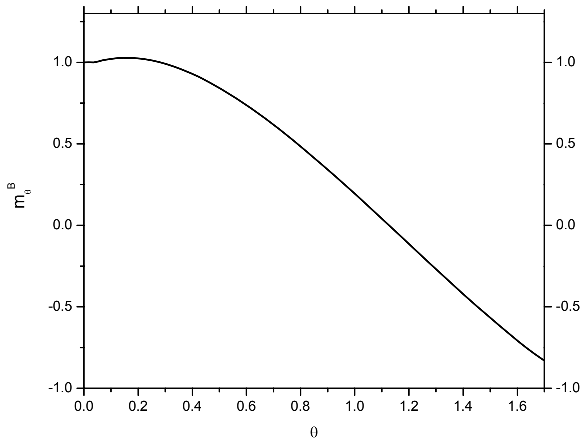
- Homogeneous slope:

- In a homogeneous slope the soil parameters will be constant within the slice
- In such conditions, we expect m_θ to be a smooth curve with a maximum of one when $\theta=0$

Enhanced Limit Equilibrium Technique
Internal plane and m_θ

- Non-Homogeneous slope:
- In a non-homogeneous slope, m_θ curve may not be smooth
- After the slice is solved, the variation of m_θ need to be checked and if necessary the geometry angles may be changed to satisfy that m_θ does not exceed one any where

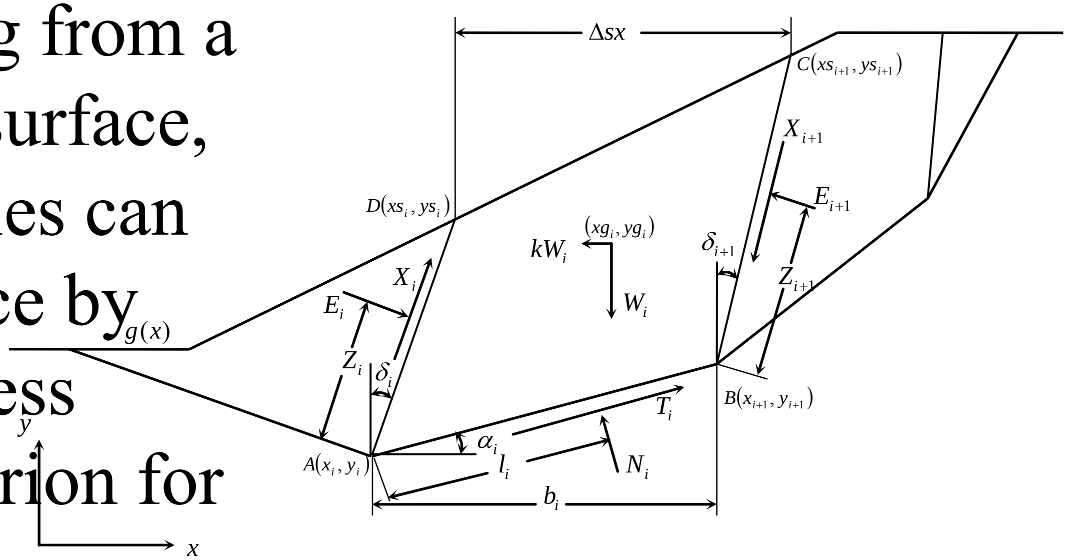




Seismic Slope Safety Assessment

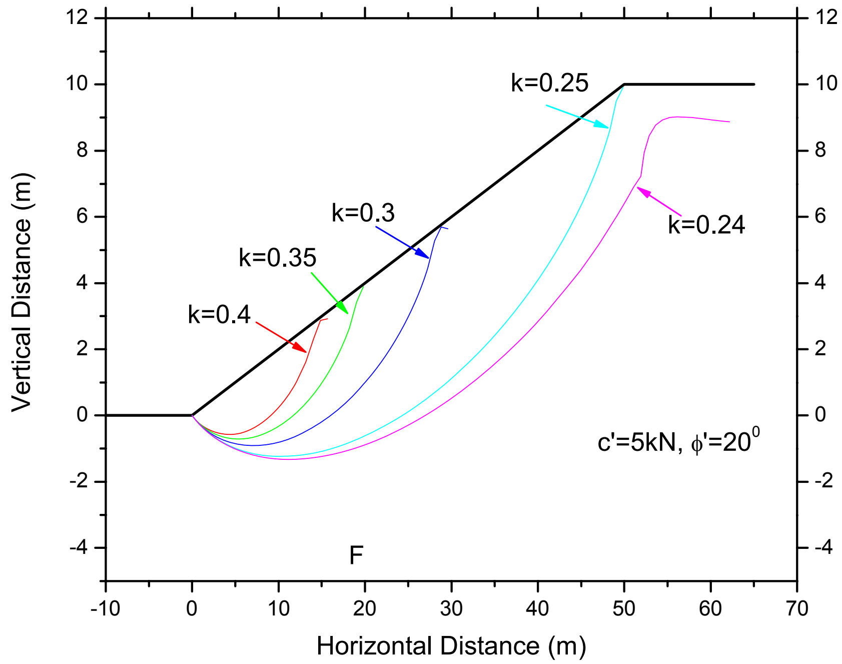
Enhanced Limit Equilibrium Technique

- Therefore, starting from a point on the free surface, the geometry angles can be determined slice by slice to satisfy stress acceptability criterion for a given k_c.
- Then check whether the slip surface satisfies the kinematic acceptability.



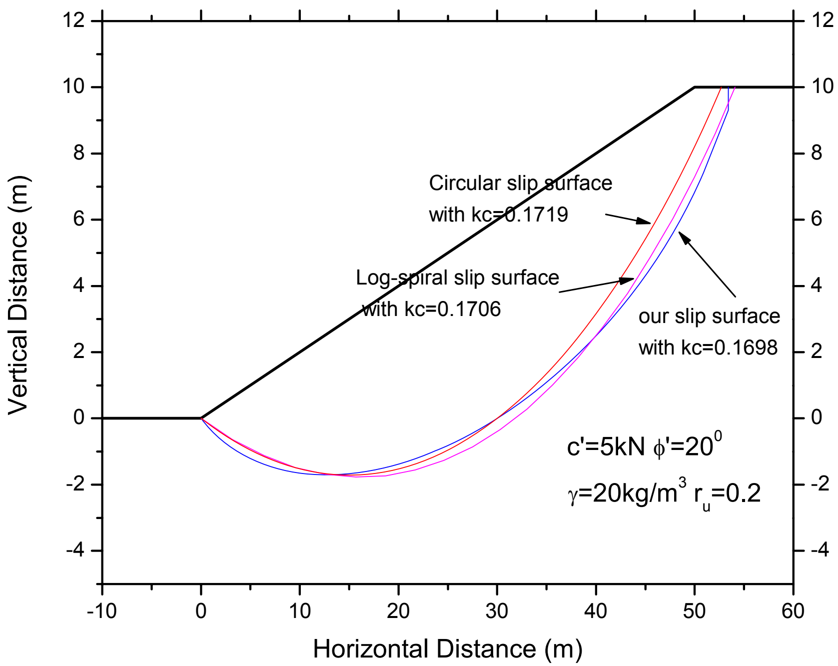
Acceptable slip surfaces (homogenous)

- Any value of $k=k_c$ can produce a slip surface
- The slip surface with a large k ended within the slope but acceptable.
- The slip surface with a small value of k did not converge towards the crest of the slope. These are not acceptable slip surfaces.

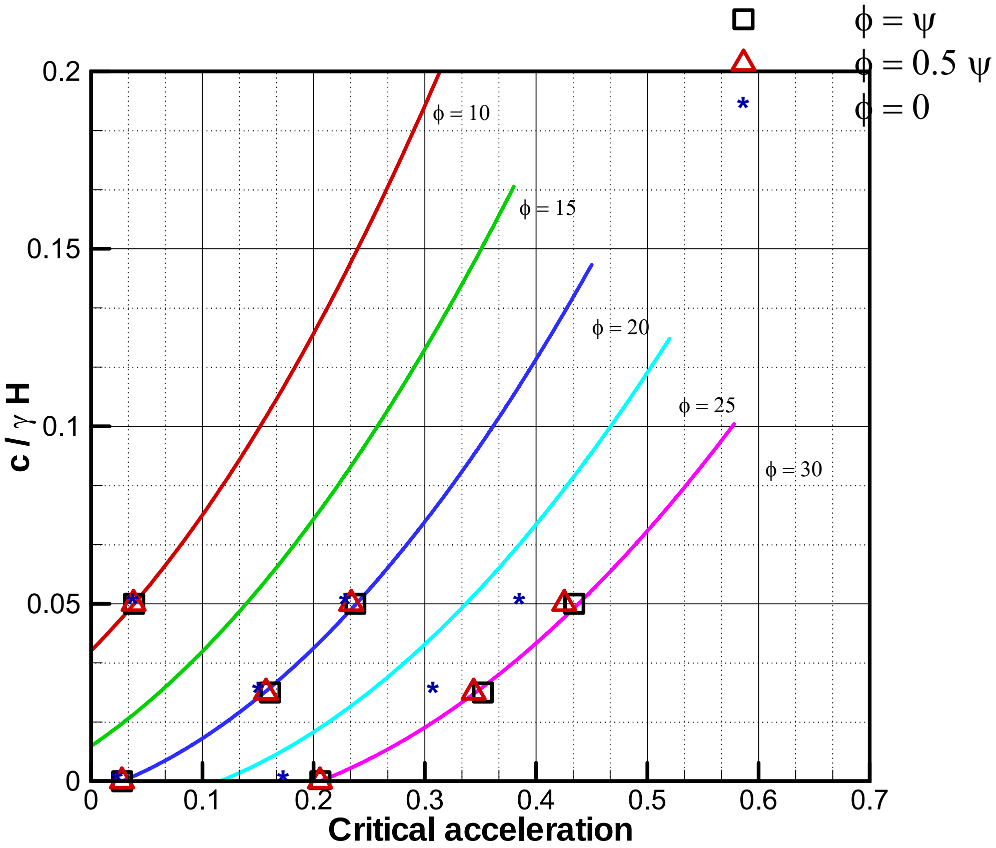


Examples: homogeneous slope

- The critical acceleration obtained is comparatively smaller than the circular arc and the log-spiral surfaces
- the critical surface found is neither a log-spiral nor a circular arc but close to both of them

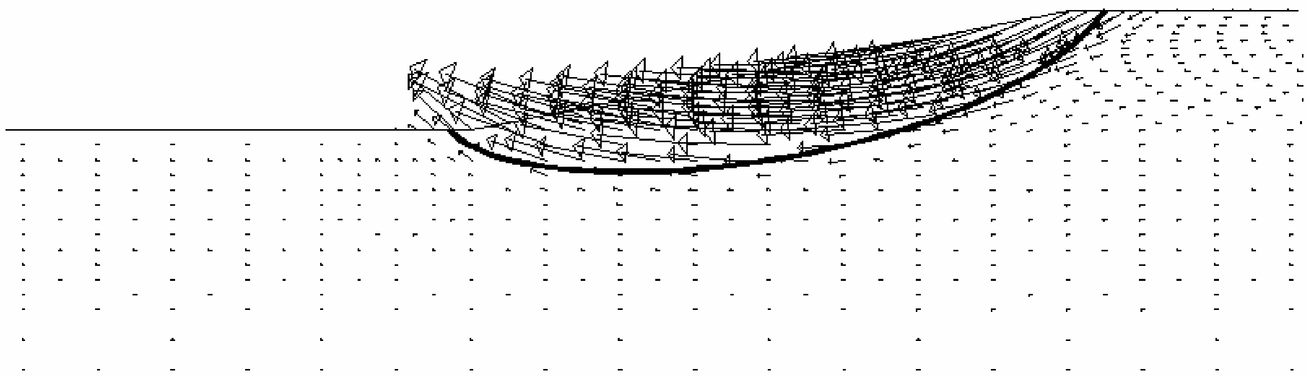


Verification of critical acceleration by FE method for 1/3 slope, Tan & Sarma(2008)

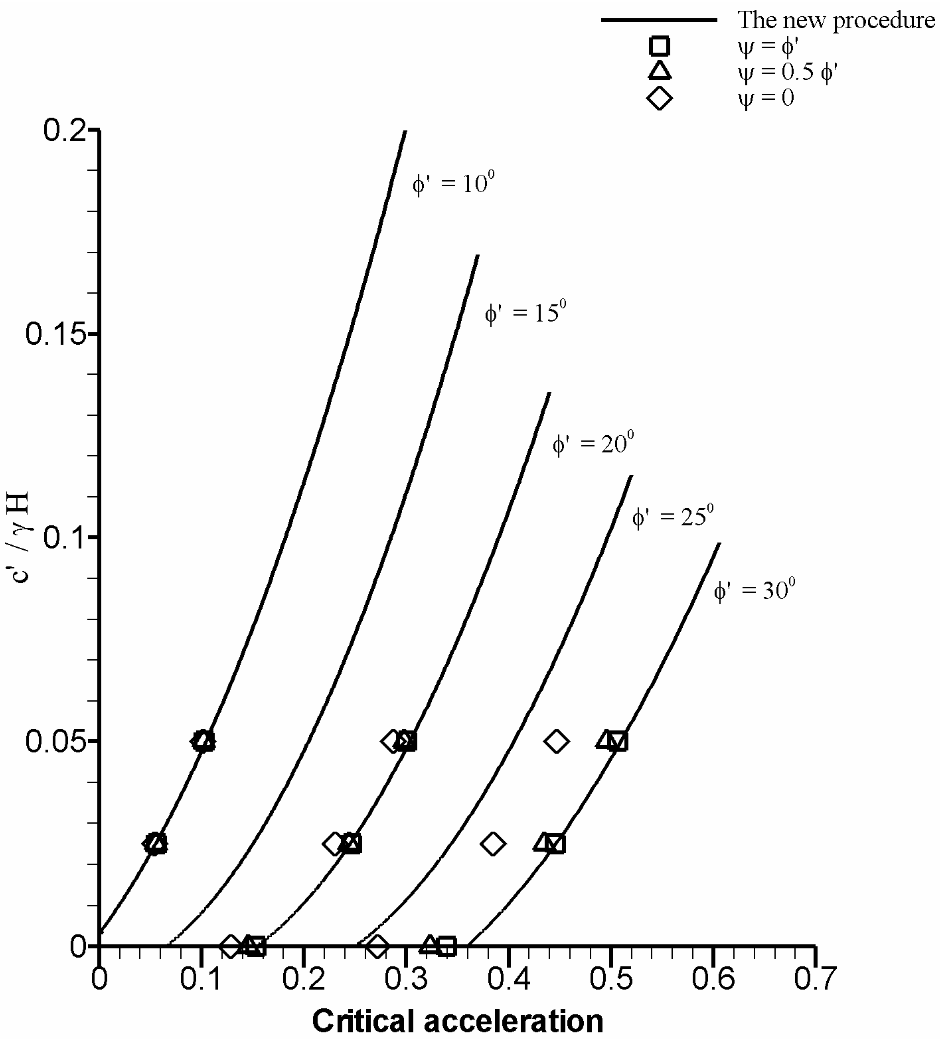


Verification of critical slip surface by FE method for 1/5 slope

$c' = 5 kN/m^2$, $\phi' = 10^0$, $\psi' = \phi'$, Tan & Sarma(2008)

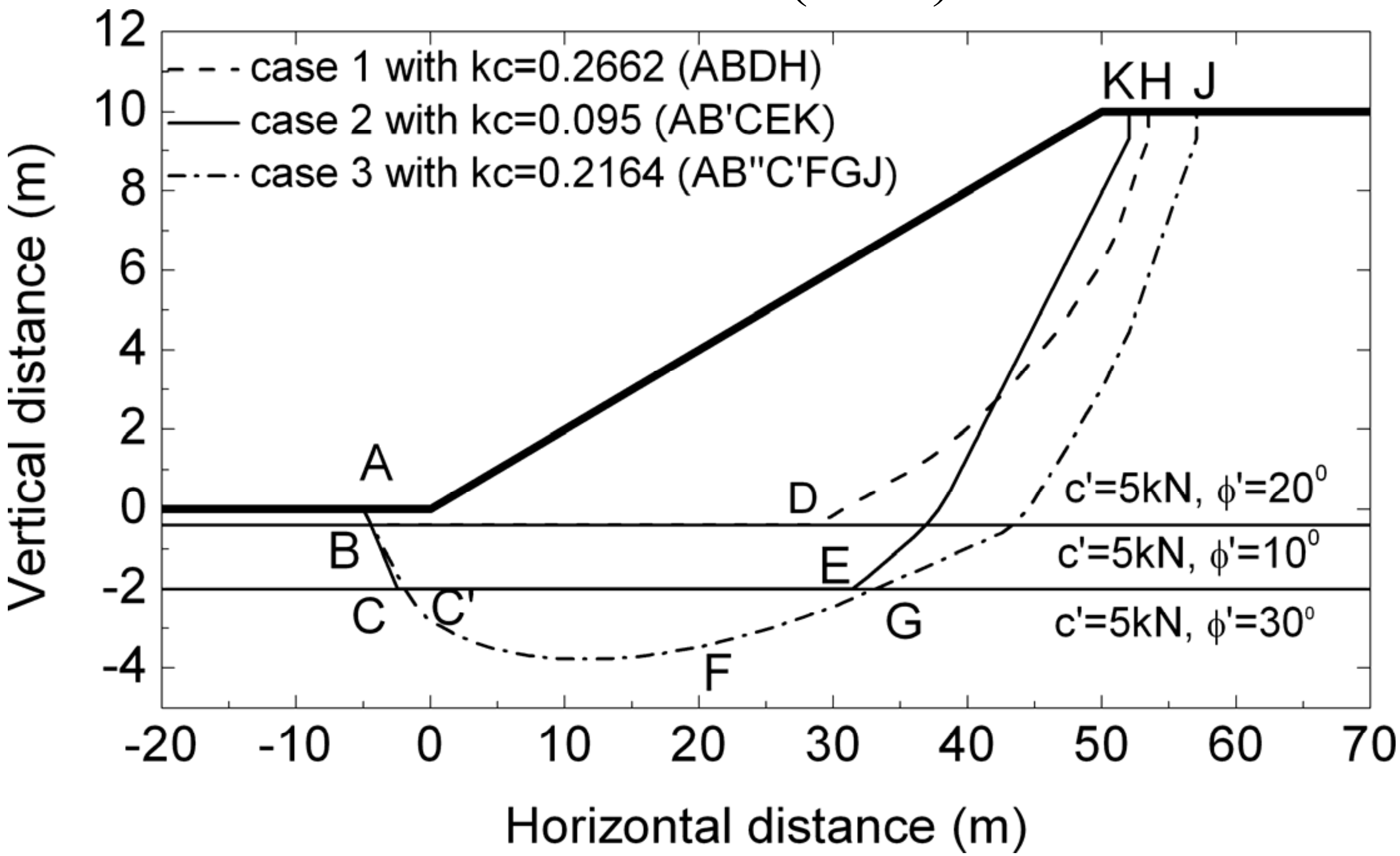


Verification of critical acceleration by FE methods for 1/5 slope, Tan & Sarma(2008)

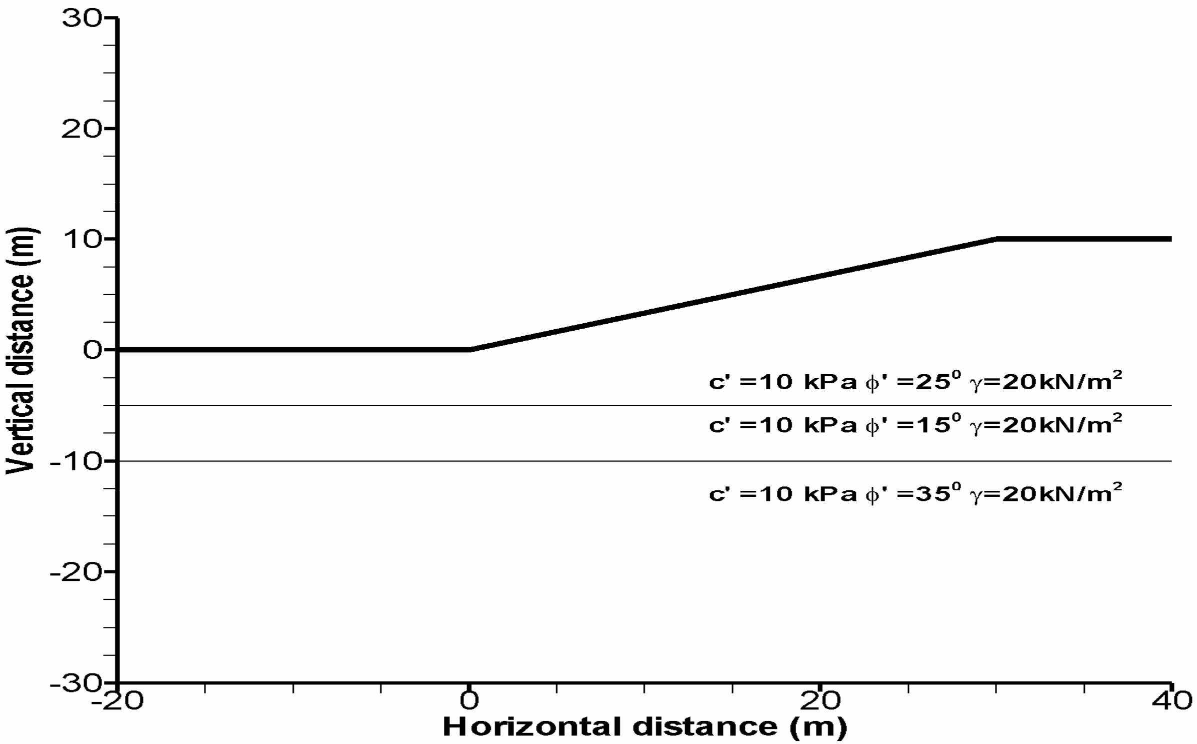


Non-homogeneous slope
Slip surface Path,

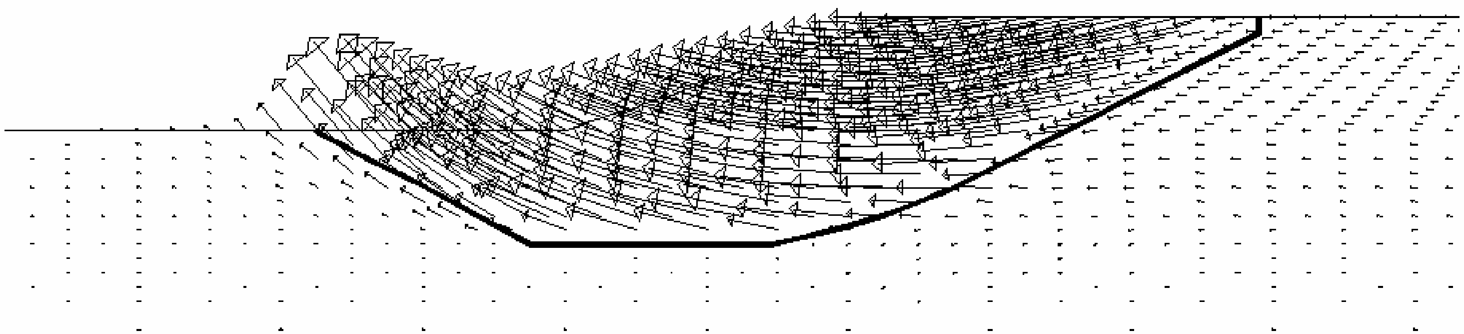
Sarma& Tan (2006)



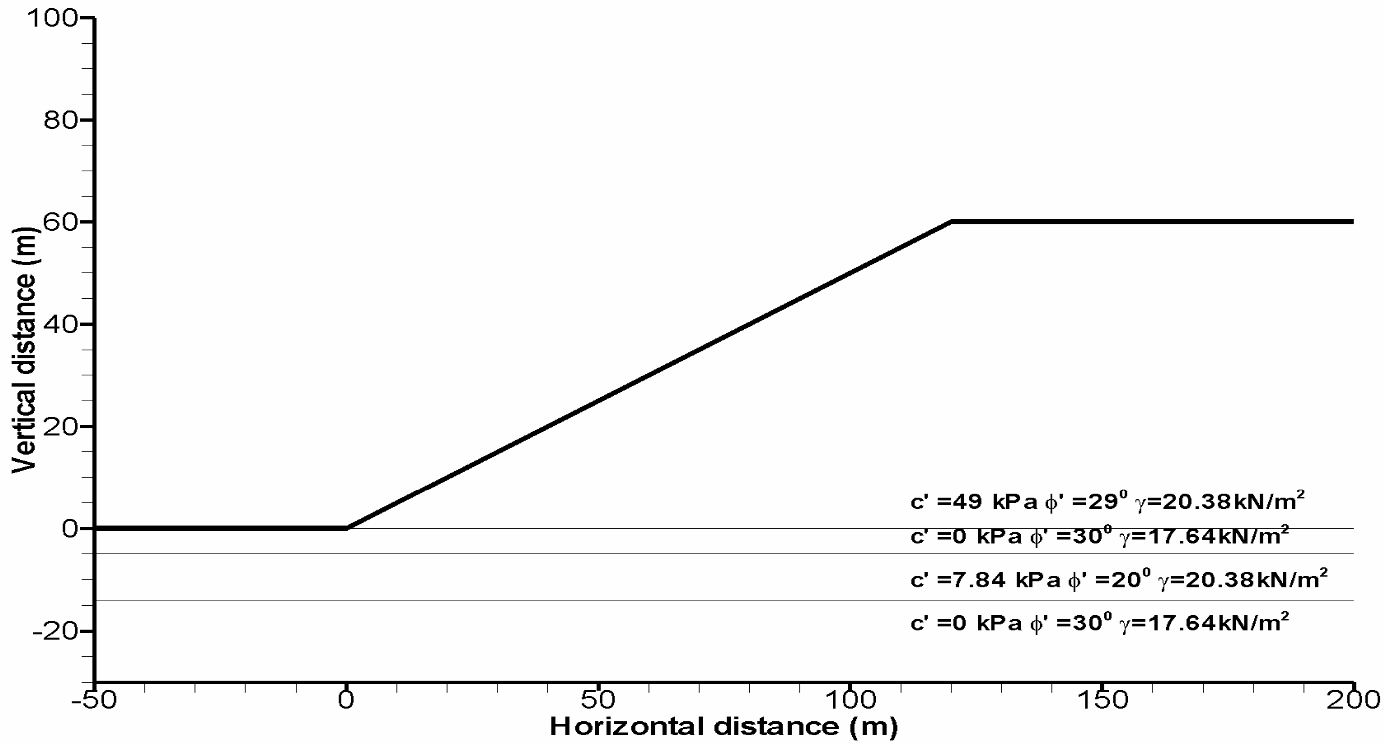
Non-homogeneous slope



Non-homogeneous slope
FE Verification
k_c=0.26 vs. 0.254(FE), Tan & Sarma(2008)

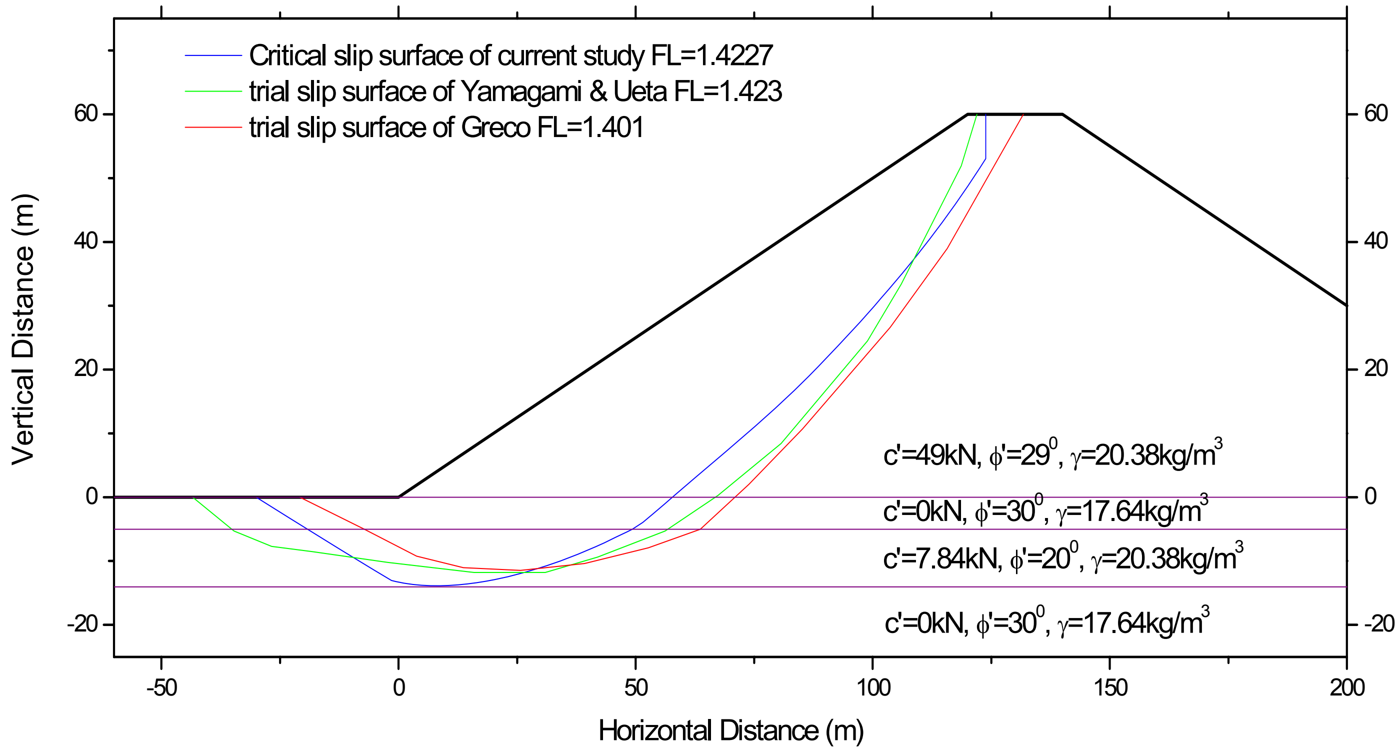


Non-homogeneous slope



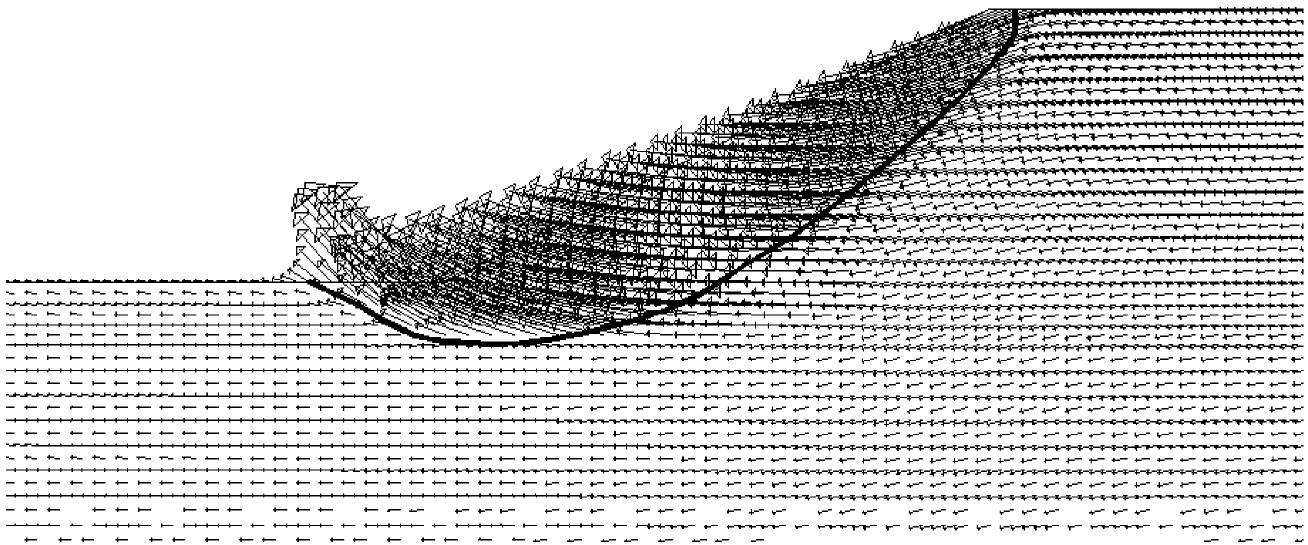
Example: non-homogenous slope

Sarma & Tan (2006)



Non-homogeneous slope

Tan & Sarma(2008)



Seismic Slope Safety Assessment

Critical Slip Surface

- Alternative methods to Determine Critical slip surface

    Pattern Search Method

    Random Search Method

    Calculus based method

- These may not satisfy acceptability Criteria

Seismic Slope Safety Assessment

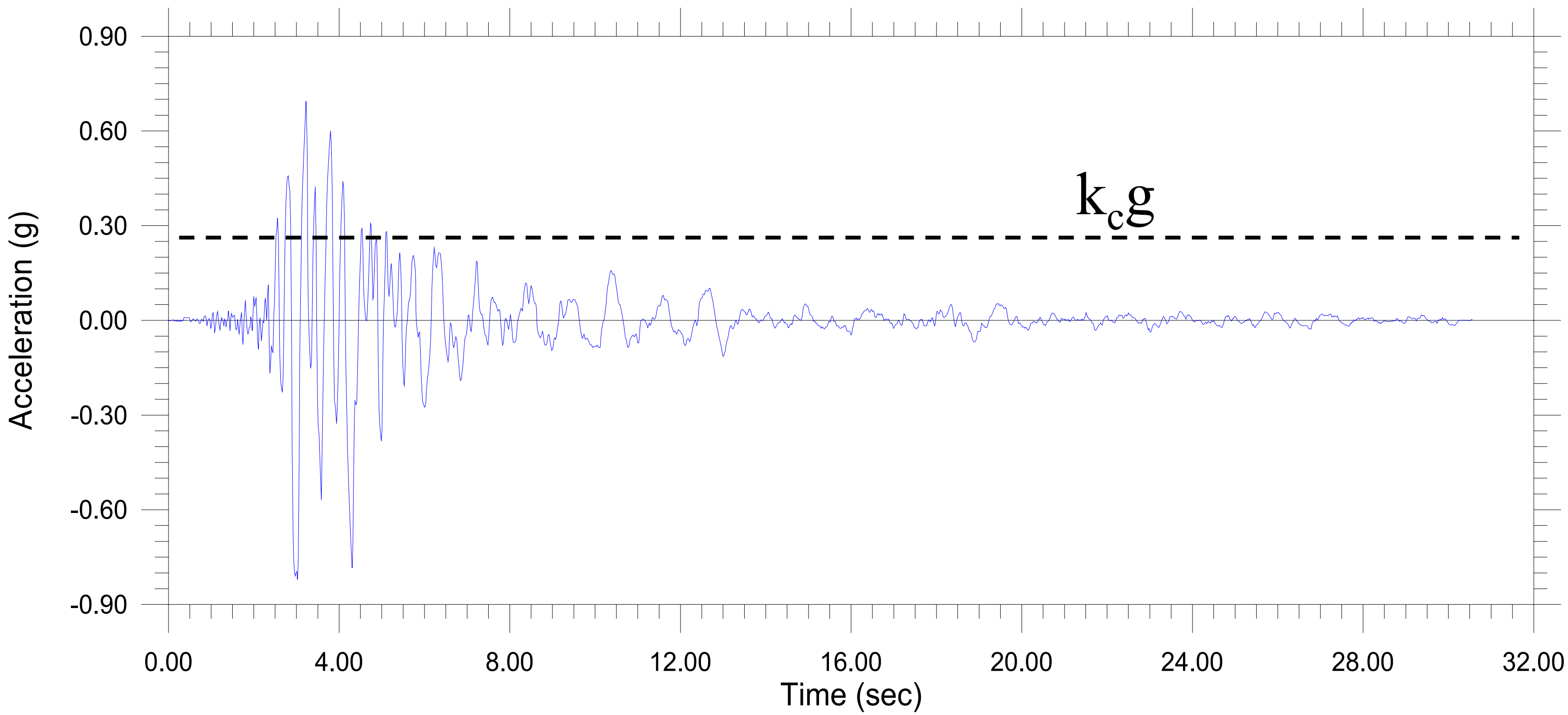
Seismic Displacement

- When the resistance is not sufficient to withstand the load, the consequence is sliding displacements.

Seismic Slope Safety Assessment

Seismic Displacement

- Strong Earthquake may produce Peak Ground acceleration $k_m g > k_c g$
- Slip surface is formed and the mass of soil slides downhill.
- How much will it slide depends on many factors

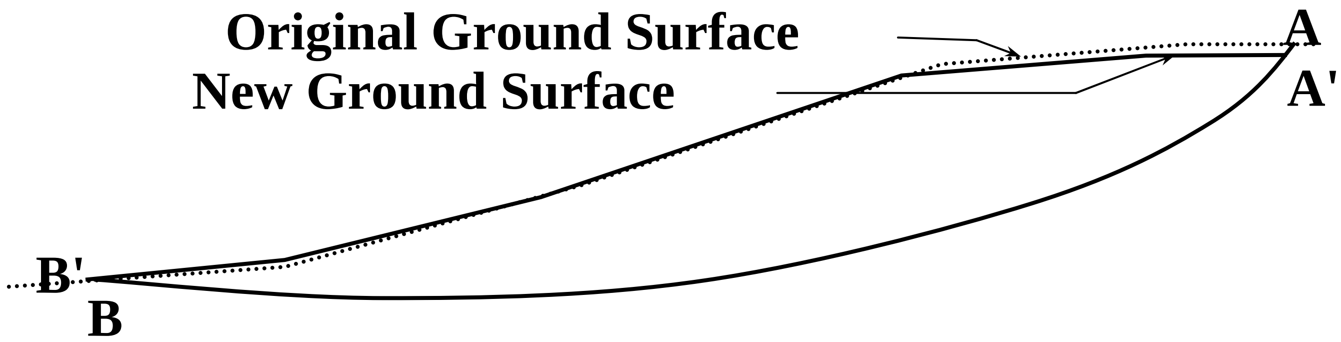


0.90
0.60
0.30
0.00
-0.30
-0.60
-0.90
Acceleration (g)
k_cg
0.00
4.00
8.00
12.00
16.00
20.00
24.00
28.00
32.00
Time (sec)

Seismic Slope Safety Assessment

SAFETY CRITERION

Seismic Displacement



<u>Relative Displacements of the sliding mass.</u>

(AA' or BB' represents the relative displacement- Note that AA' and BB' are not equal)

Seismic Slope Safety Assessment

Seismic Displacement

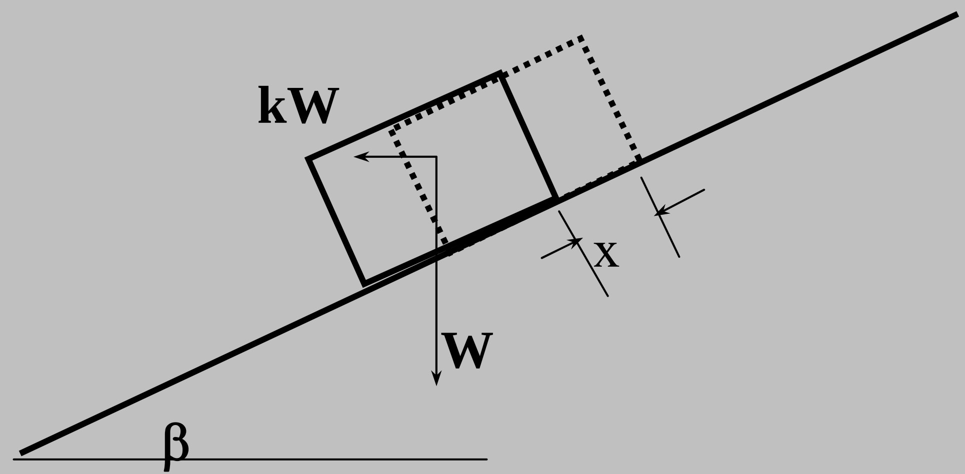
- Factors affecting displacement
  - k_c/k_m ratio
  - k_m or v_{max}
  - Duration of acceleration pulses
  - Number of pulses

  --------------------------------Sarma & Kourkoulis (2004)

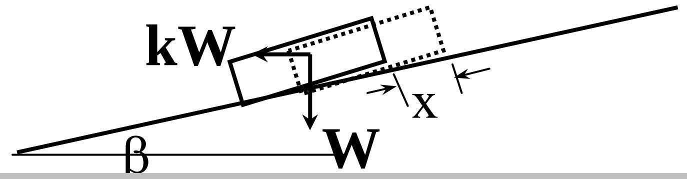
  - Change of strength parameters due to displacement
  - Change of Geometry of slide

Seismic Slope Safety Assessment
Seismic Displacement

- Newmark (1965) sliding block technique
  - Simplified analysis
  - Valid for small displacements



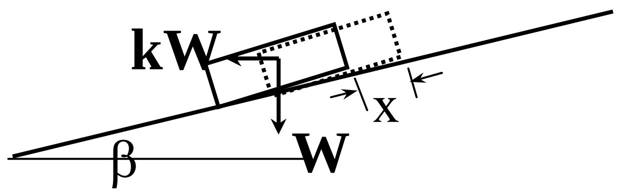
Seismic Slope Safety Assessment



<u>Equivalent Sliding Block Model</u>
(x represents the relative displacement)

The mechanics of the sliding block
D= Driving Force down the slope
= W (sinβ + k cosβ)
R= Resisting Force
= [W(cosβ - k sinβ) -U] tan ϕ' +c'L
where
U= Force due to pore water pressure
and
L= Length of the block (represents the length of the slip surface)

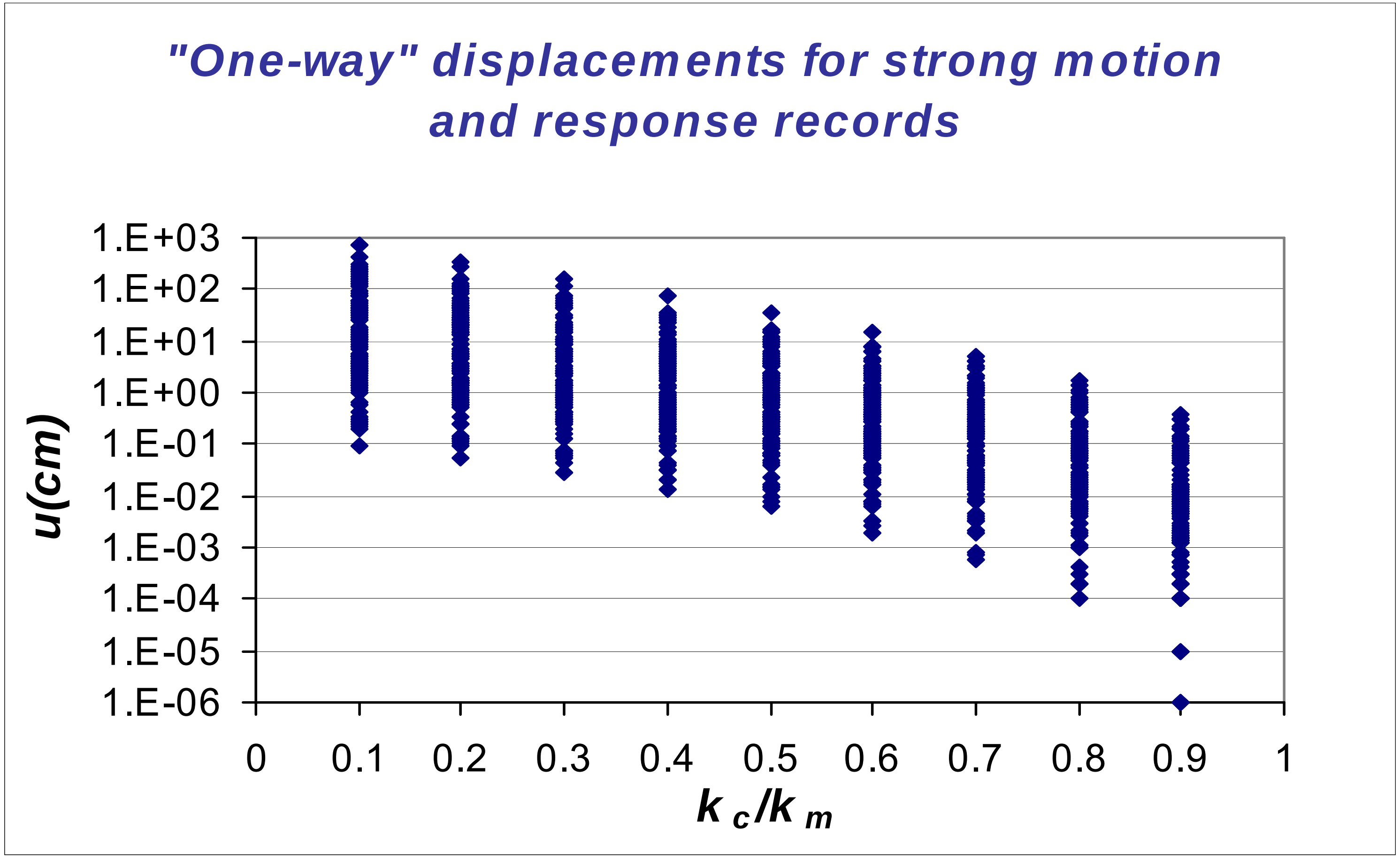
Seismic Slope Safety Assessment



$$\frac{W}{g}\ddot{x} = D - R = W\frac{\cos(\phi' - \beta)}{\cos\phi'}(k - k_c)$$

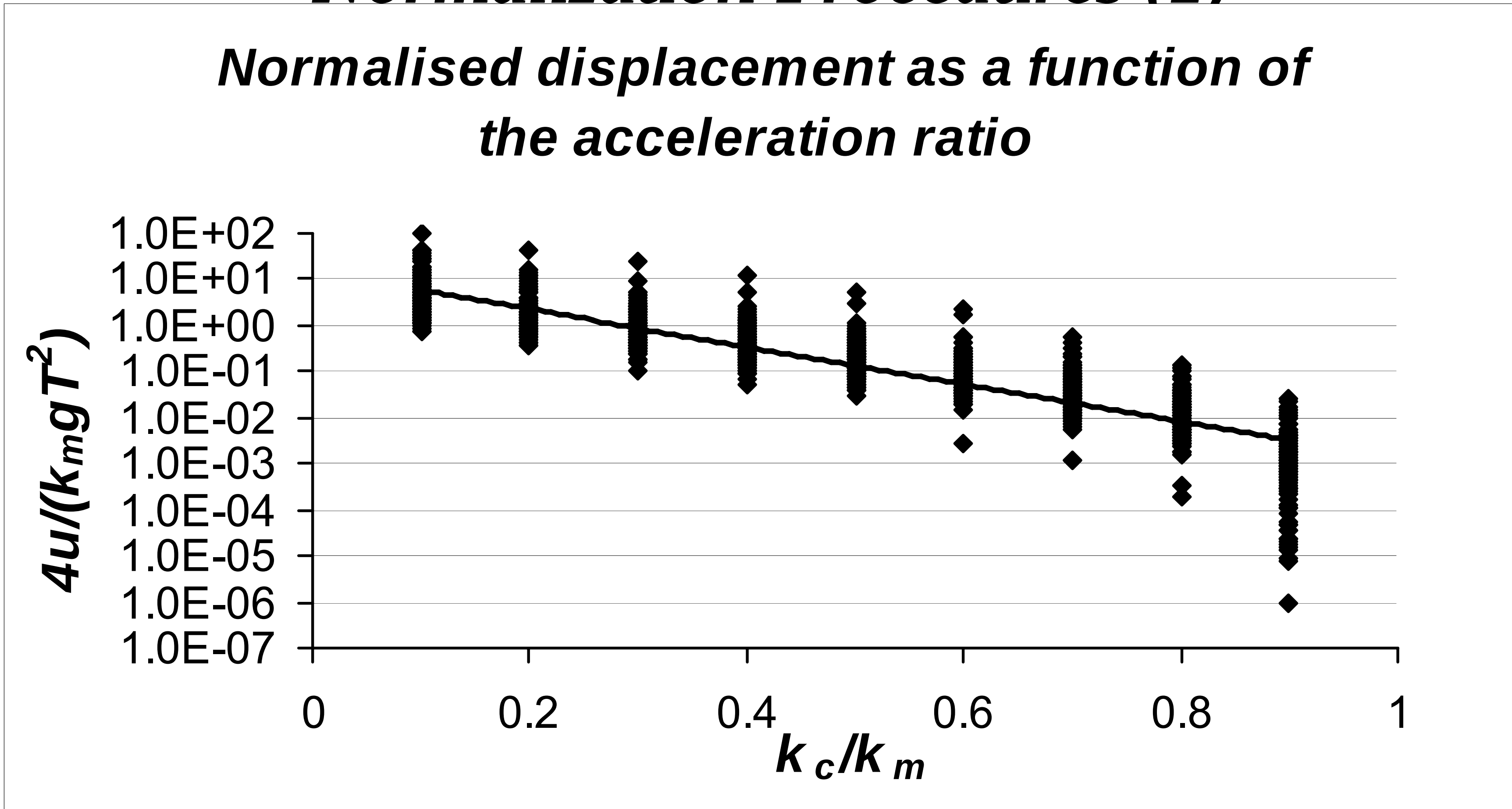
$$k_c = \tan(\phi' - \beta) - U\frac{\sin\phi'}{\cos(\phi' - \beta)} + c'L\frac{\cos\phi'}{\cos(\phi' - \beta)}$$

Accuracy of the Sliding Block Model



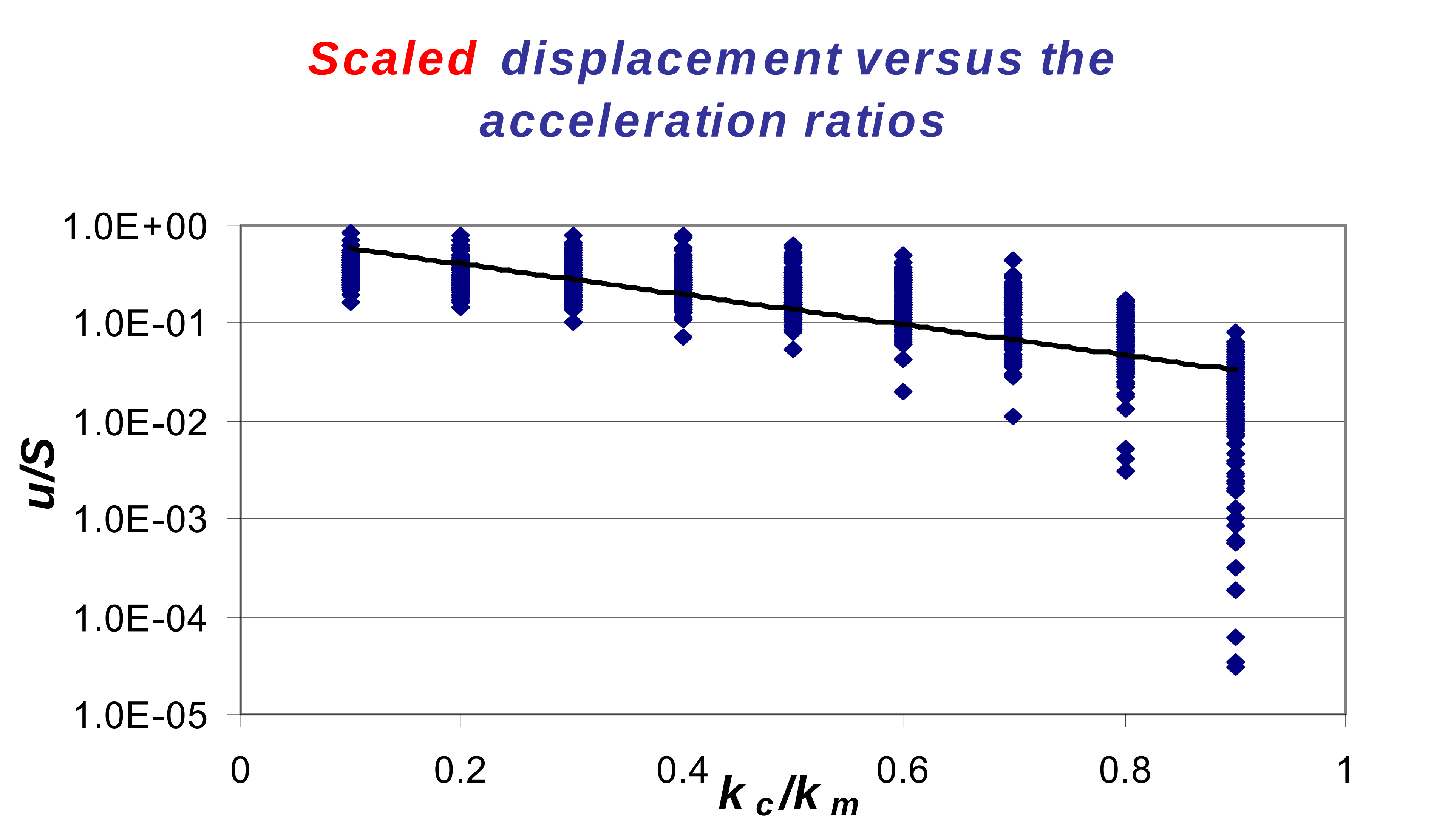
- Scatter of ***4 orders of Magnitude***
- Indicative that the knowledge of the k_c/k_m ratio is inadequate to predict the displacement, Sarma& Kourkoulis (2004)

Normalization Procedures (1)



- **Initial Scatter Reduced to about *2 orders of Magnitude***

Normalization Procedures (2)



Initial Scatter Reduced to only 1 *order of Magnitude*

Seismic Displacements

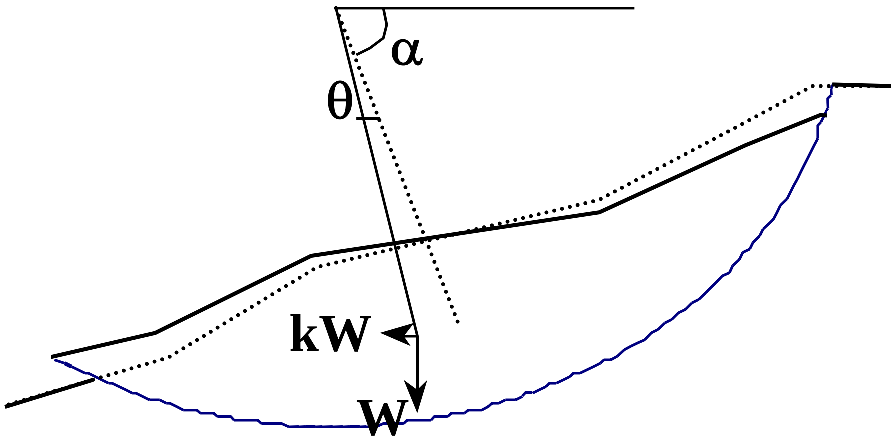
$$\log u \text{ (cm)} = C_0 + C_1 \log v_{max} + C_2 \log T_{dn} + C_3 \log n$$

(T_{dn} = dur/No)

C_0 –C_3 : Coefficients for the Prediction of Sliding Block Displacements. These are functions of k_c/k_m

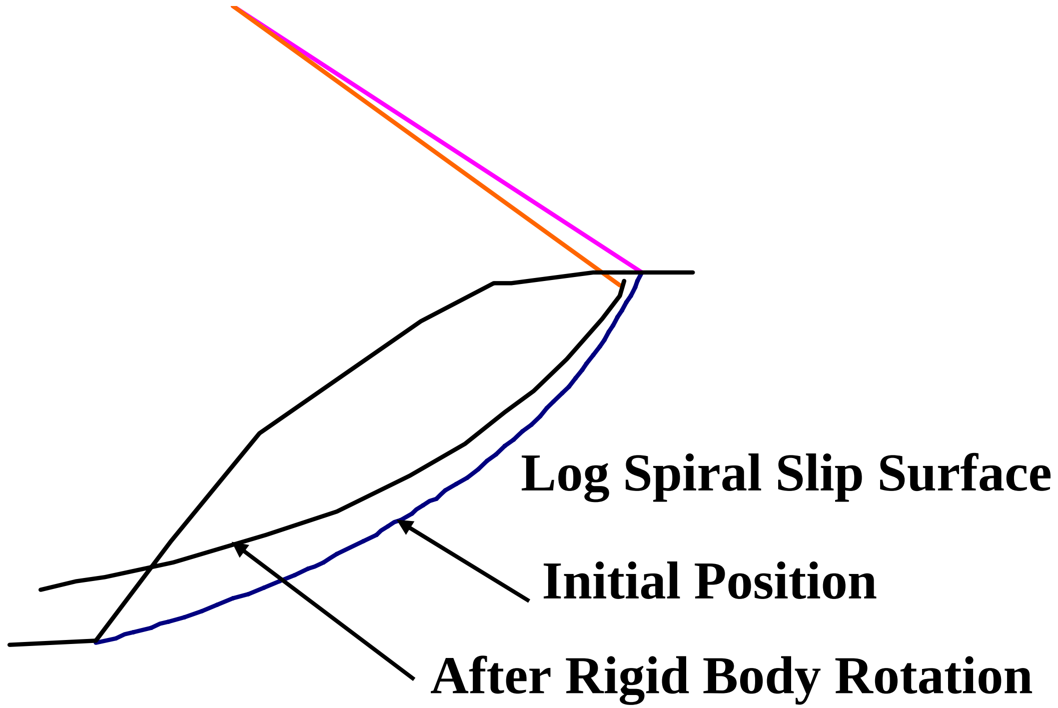
Sarma & Kourkoulis (2004)

Seismic Slope Safety Assessment



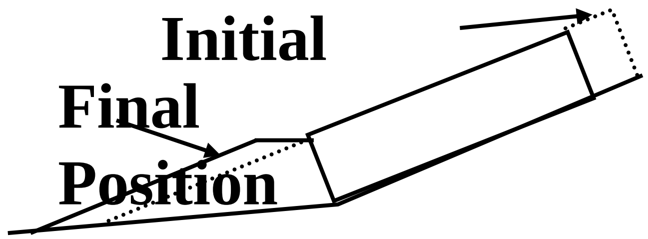
Circular arc sliding displacements

Seismic Slope Safety Assessment



<u>Rigid body rotation about the centre of the log- spiral.</u>
[Note how the sliding surface separates from the parent body]

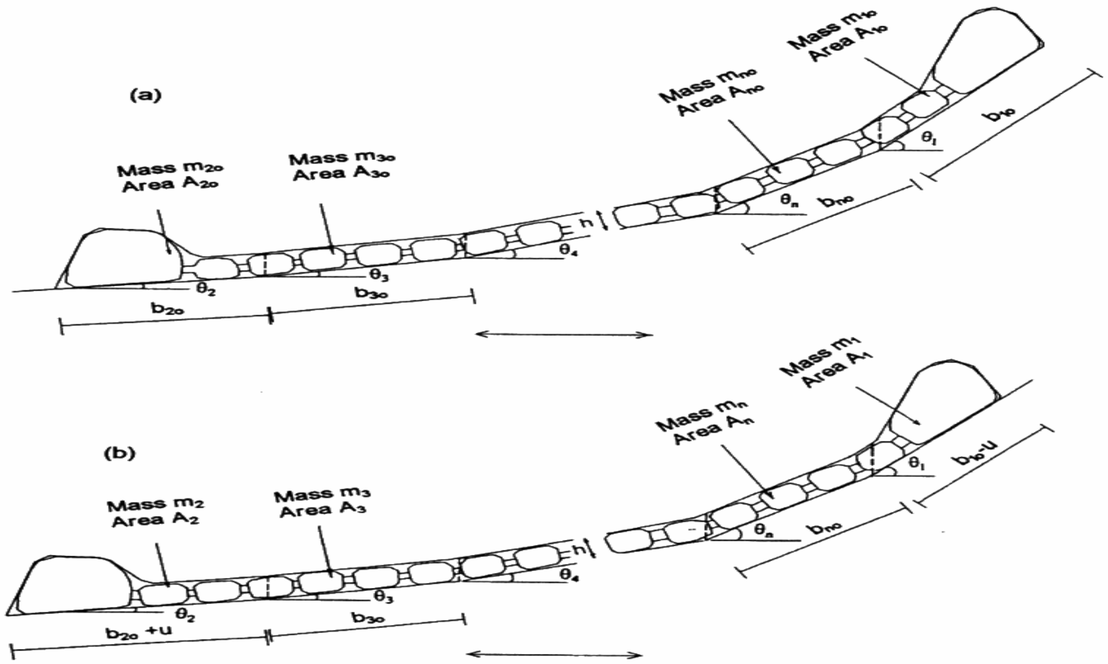
Seismic Slope Safety Assessment



Ambraseys & Srbulov (1995) Two-block sliding model with transfer of mass from the higher block to the lower block.

Two blocks slide on two different planes. Internal deformations occur along the interslice surface. Mass from the top block transfers to the lower block. As the displacement increases, the lower block grows providing more resistance to sliding.

Seismic Slope Safety Assessment



<u>**Sliding Chain model (after Stamatopoulos 1996)**</u>

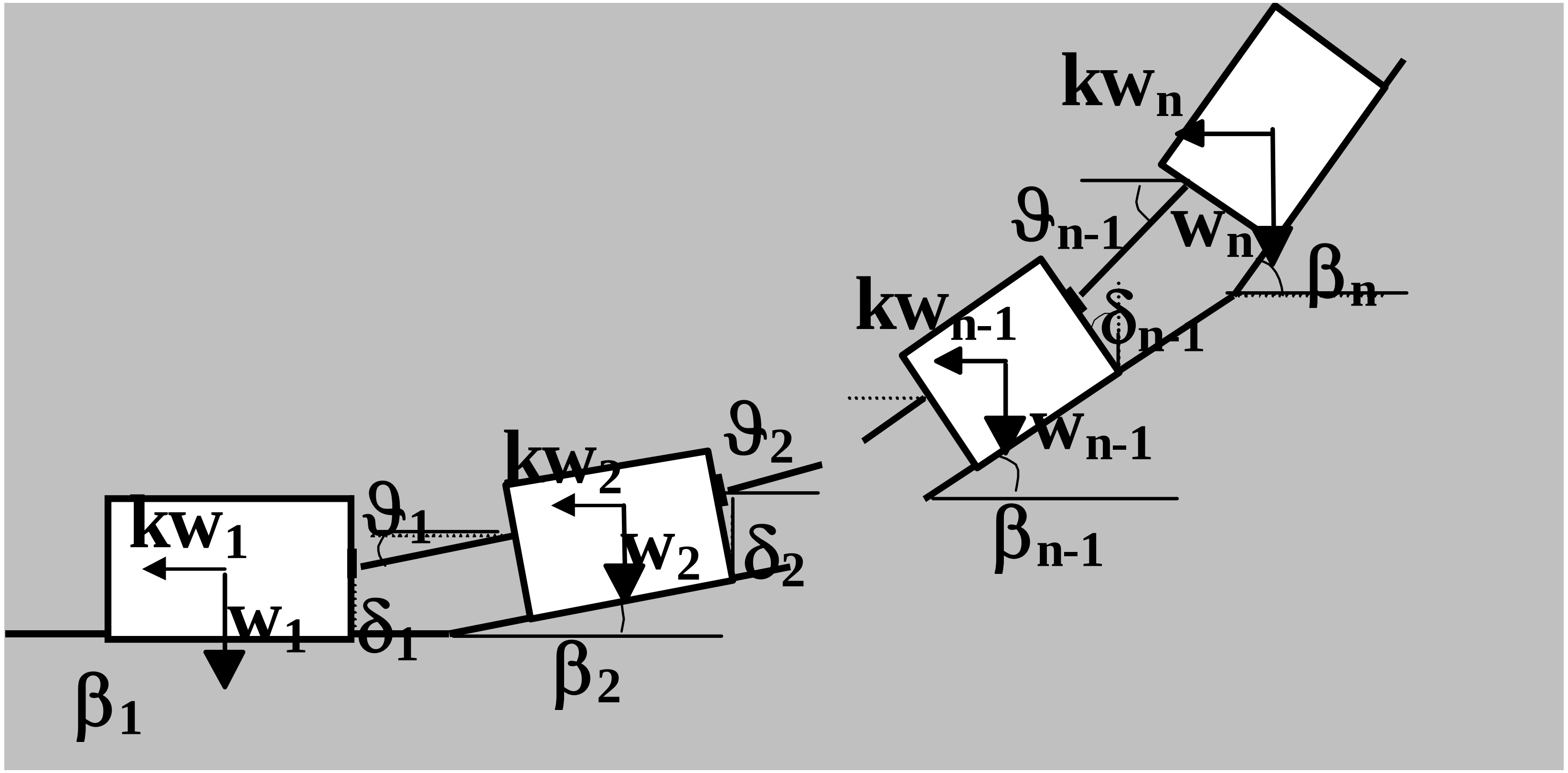
Model of a chain of linked blocks. This allows transfer of mass from the top-most block to the bottom-most block, the intermediate blocks remain unaffected. No internal deformations are considered, the chain-links being connected together by frictionless hinges. The chain is considered to be of uniform cross section.

Seismic Slope Safety Assessment

- Sarma and Chlimintzas (2001)
- Multi-block sliding Model
  - Rigid rods on sliders connects blocks
  - Mass Transfer from Higher block to Lower block
  - Change of Geometry
  - Valid for large displacements

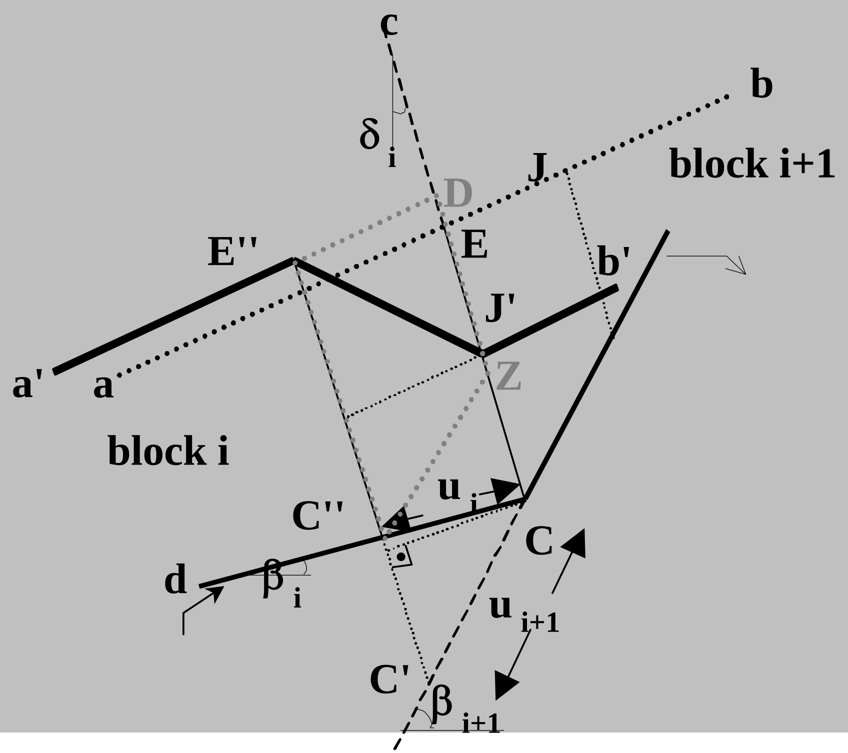
Seismic Slope Safety Assessment

Seismic Displacement



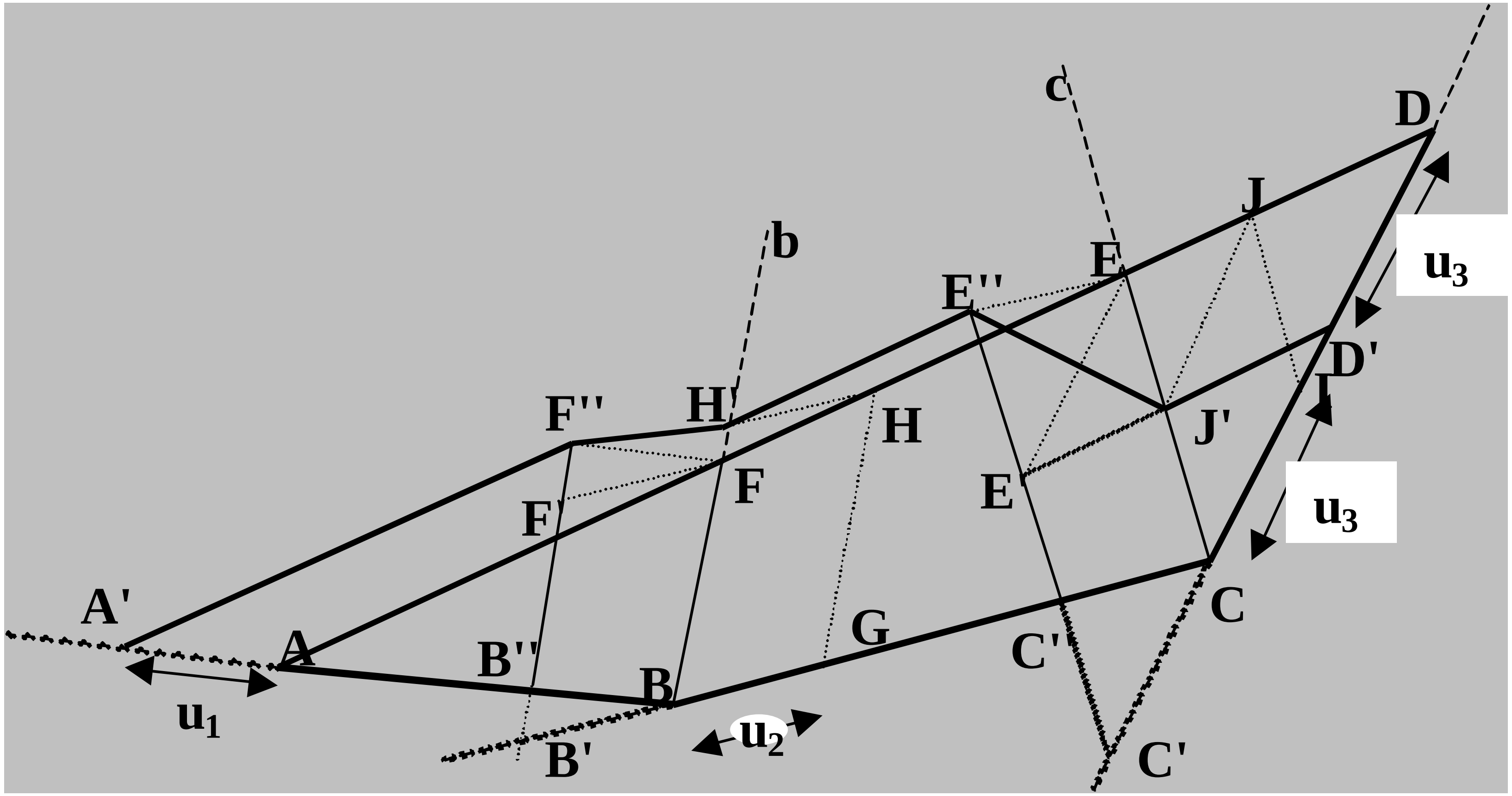
Seismic Slope Safety Assessment
Seismic Displacement

- Mass Transfer Model



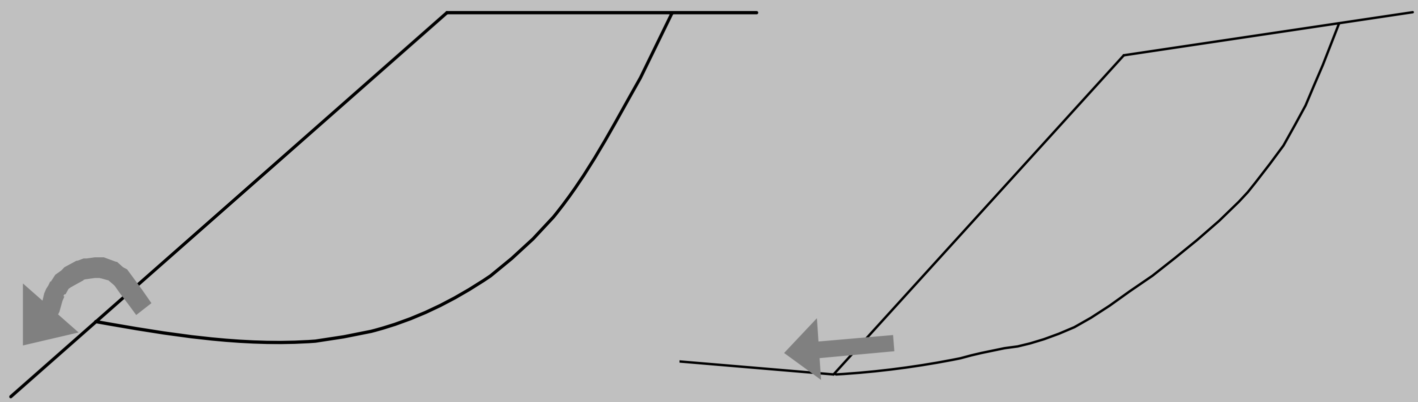
Seismic Slope Safety Assessment

Seismic Displacement



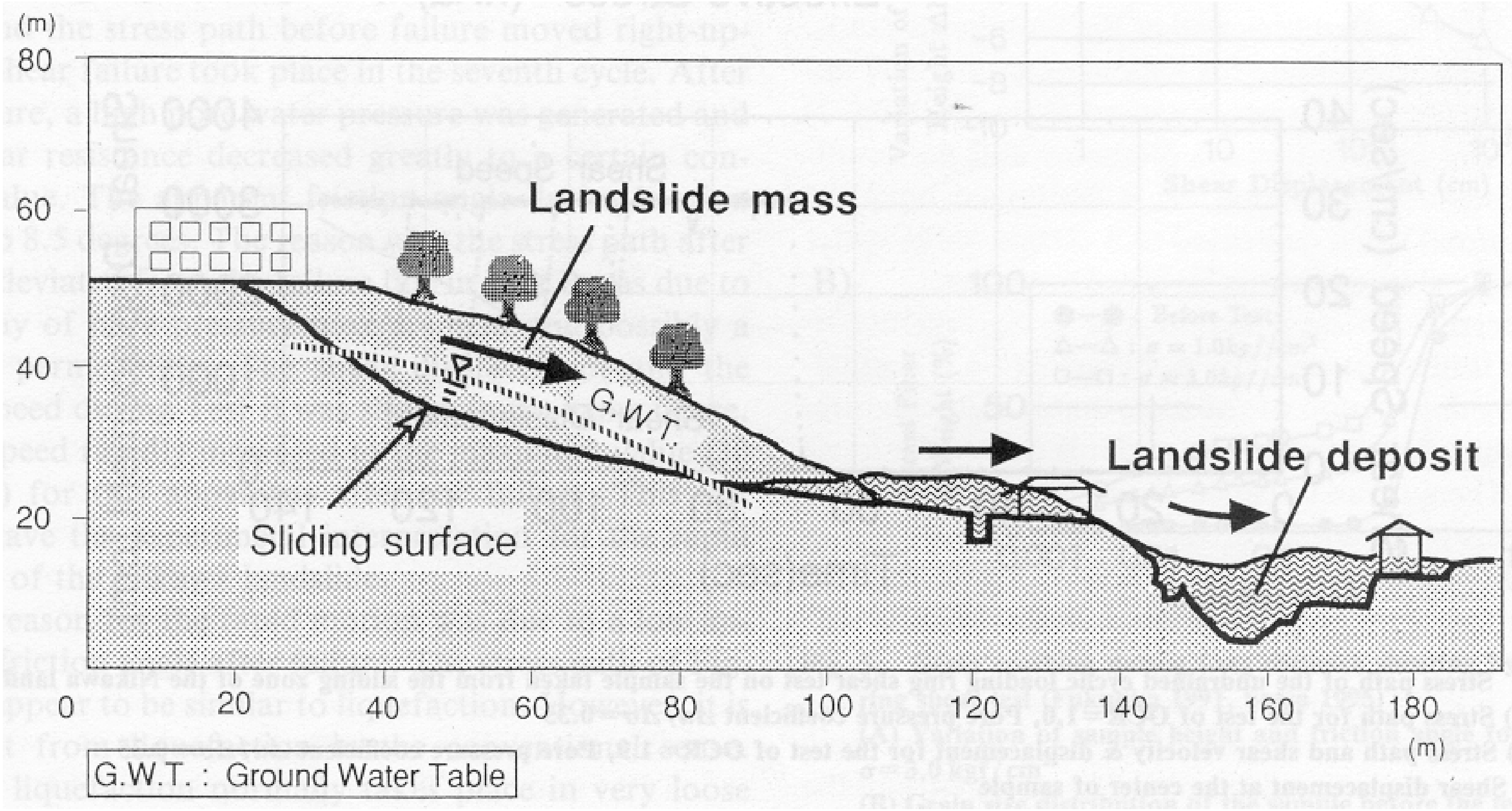
Seismic Slope Safety Assessment
Seismic Displacement

- The Toe Problem
- Sliding material may be lost or it may provide resistance



Seismic Slope Safety Assessment

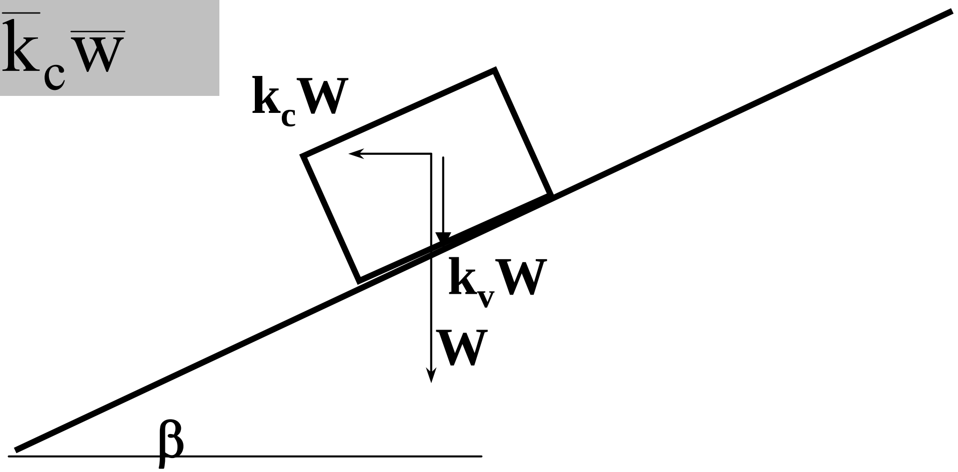
Kobe Earthquake, Nikawa Landslide



Seismic Slope Safety Assessment

vertical acceleration

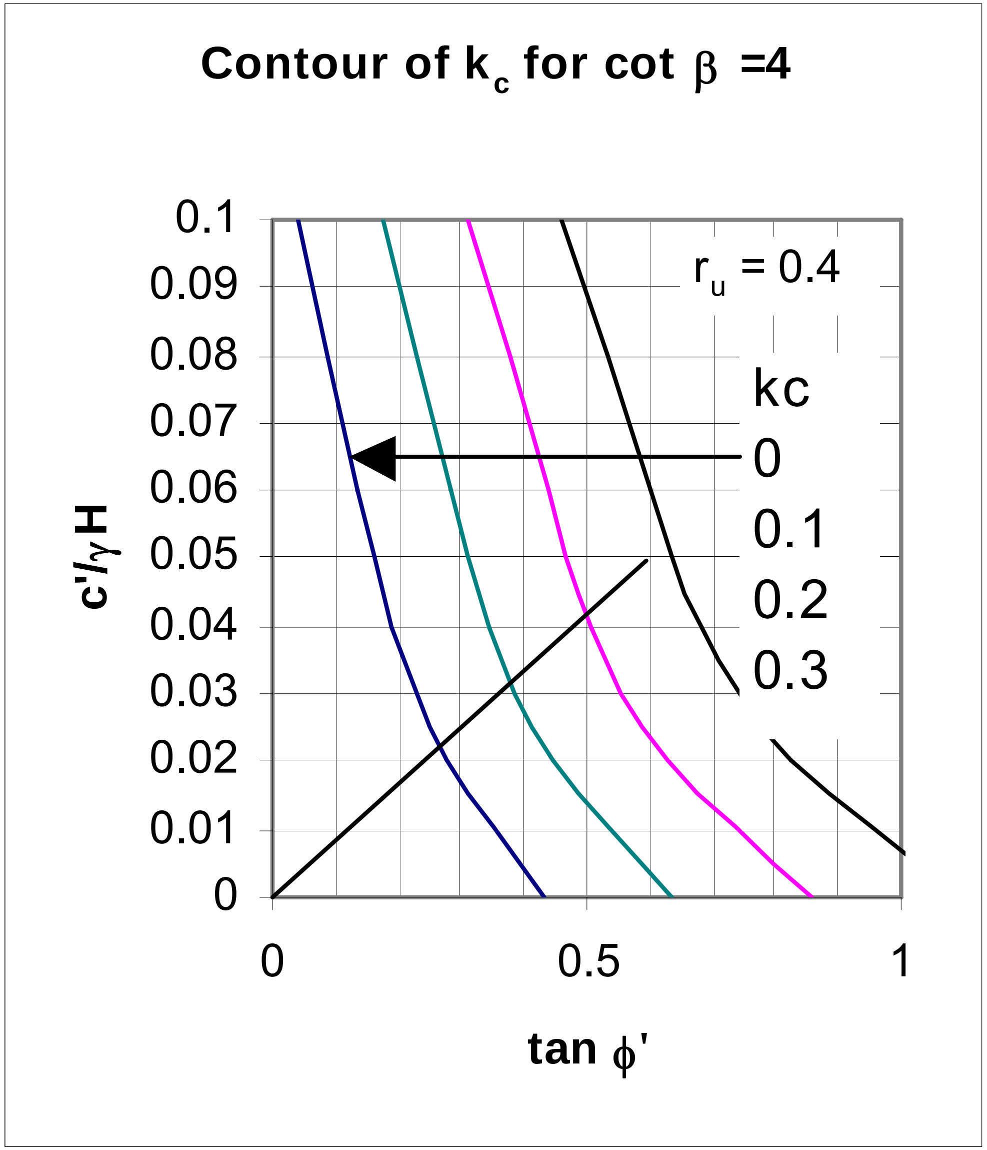
- $$\overline{w} = w(1 + k_v)$$
  $$\overline{k}_c = \frac{k_c}{(1 + k_v)}$$
  $$k_c w = \overline{k}_c \overline{w}$$



Seismic Slope Safety Assessment

Vertical Acceleration

- Note that Inertia force is positive acting downward
- Increase the unit weight of all materials including that of water. $\bar{\gamma} = \gamma(1 + k_v)$
- Compute modified critical horizontal acceleration $\bar{k}_c$ with changed weights.
- Compute the true critical horizontal acceleration. $k_c = \bar{k}_c(1 + k_v)$



Homogeneous slope (1/4 Gradient)

PWP parameter $r_u = u/\gamma H$

Seismic Slope Safety Assessment

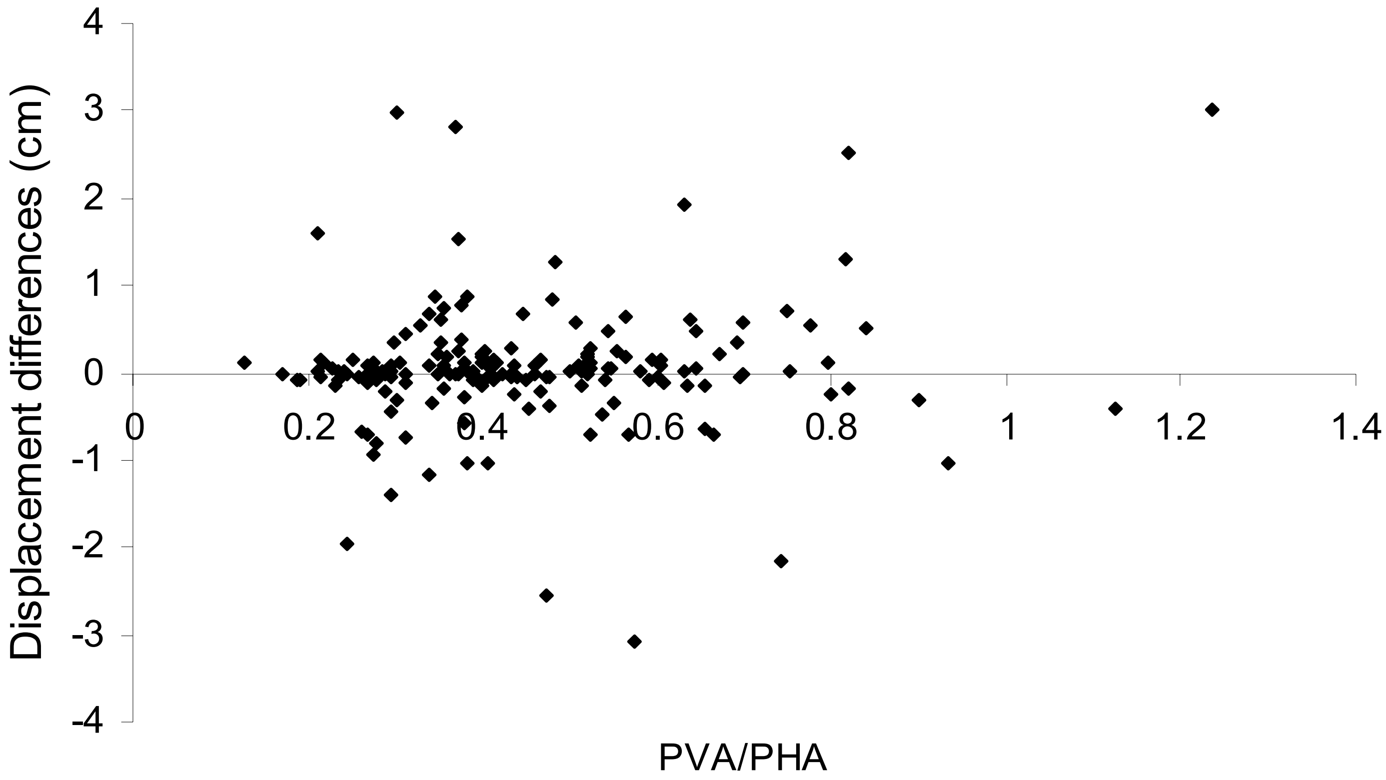
Vertical Acceleration

- For cohesive soils:
  - Effect of increasing the unit weight is to reduce the effect of cohesion. Therefore, critical horizontal acceleration tend to reduce.
  - Effect of modifying the critical acceleration is to increase the value for positive vertical inertia load.
  - Net effect for cohesive soils is small
- For cohesionless soils:
  - Negative vertical inertia force will always reduce the critical horizontal acceleration.

Effect of Vertical Accelerations

Displacement Differences for cohesionless slopes as a function of PVA/PHA for kc/km = 0.1,

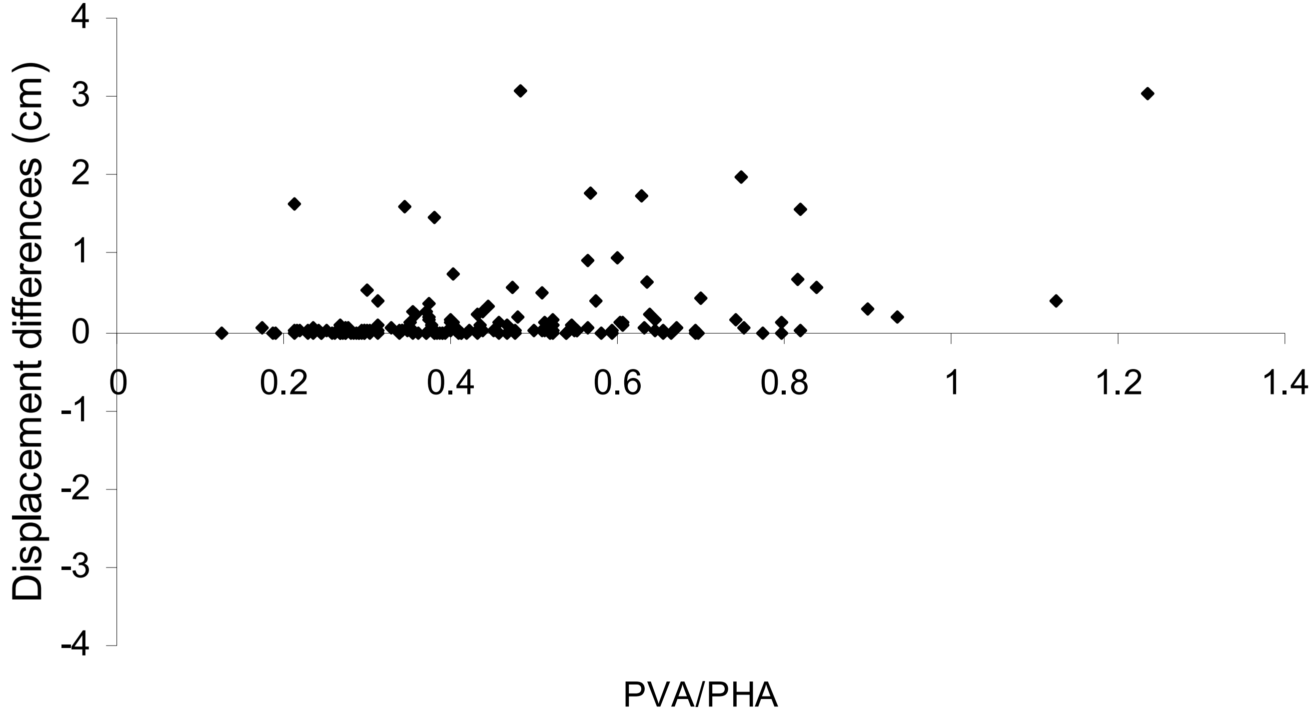
Sarma& Scorer (2009)



Effect of Vertical Accelerations

Displacement Differences for cohesionless slopes as a function of PVA/PHA for kc/km = 0.9,

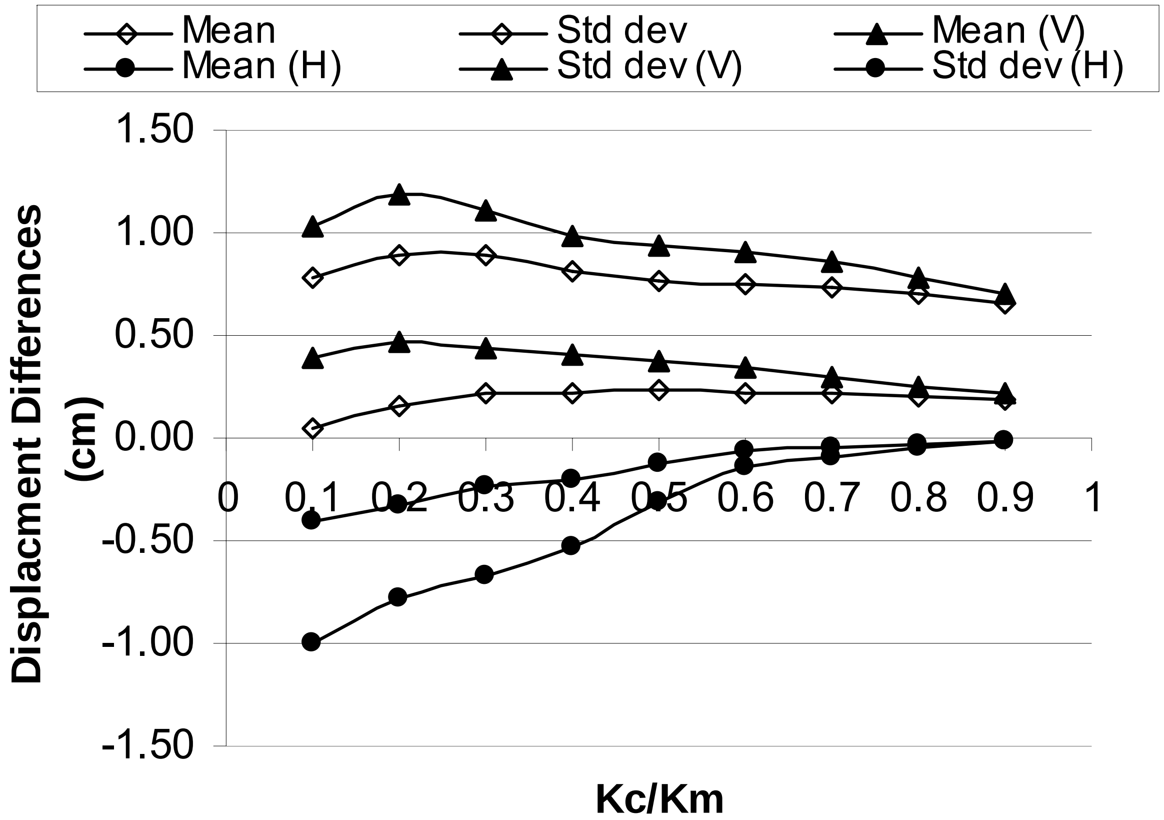
Sarma& Scorer(2009)



Effect of Vertical Accelerations

Displacement Differences as a function of kc/km,

Sarma& Scorer(2009)



Seismic Slope Safety Assessment

Analytical Approach

- Conclusions
  - The Analytical approach provides a tool for assessing slope safety during earthquakes;
  - The three stages of the analysis should be understood and applied correctly;

Seismic Slope Safety Assessment

Analytical Approach

- Conclusions:
  - The critical acceleration approach can be applied for both static and seismic situations
  - The enhanced slope stability analysis method is effective in determining both the critical accelerations and the critical surface.

Seismic Slope Safety Assessment

Analytical Approach

- Conclusions:
  - Seismic displacements govern the safety of the slope. Single block sliding on a plane is good for small displacements
  - Multi-block sliding should be considered for large displacements
  - Within the accuracy of the sliding block displacements, vertical acceleration will increase the displacements for cohesionless soils slightly but may not be important.

Acknowledgements

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